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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ERECTION OF HYDRAULIC TURBINES AND GENERATORS

By Andrew M. Komora,[1] M. Am. Soc. C. E.

Synopsis

Most engineers who have installed hydroelectric generators have discovered that certain units operate perfectly whereas others perform in a manner leaving much to be desired. Construction methods and procedures differ widely, and the writer believes that, by adhering to certain fundamentals in erection procedure, most units can be assembled so as to operate perfectly. The purpose of this paper is to offer suggestions for the perfection of such methods and procedures.

The construction procedure for assembling and erecting turbines and generators recommended in this paper is based principally on the methods used by the Tennessee Valley Authority (TVA). The installations vary from the low-head, large-diameter Kaplan-type turbines at Pickwick Landing, Tenn., to the relatively high-head Francis-type units at Hiwassee, N. C.[2] The fundamental rules apply to all types of installations and give comparable results. The steps in the erection are presented under six headings, or "stages," in the order in which they generally occur.

Stage 1. Placing and Erecting Turbine Embedded Parts

Fig. 1 illustrates cross sections through two typical turbine types, indicating the nomenclature generally used by engineers and construction personnel. Although the paper refers expressly to the typical "Kaplan" installation, the general procedure is applicable to all types.

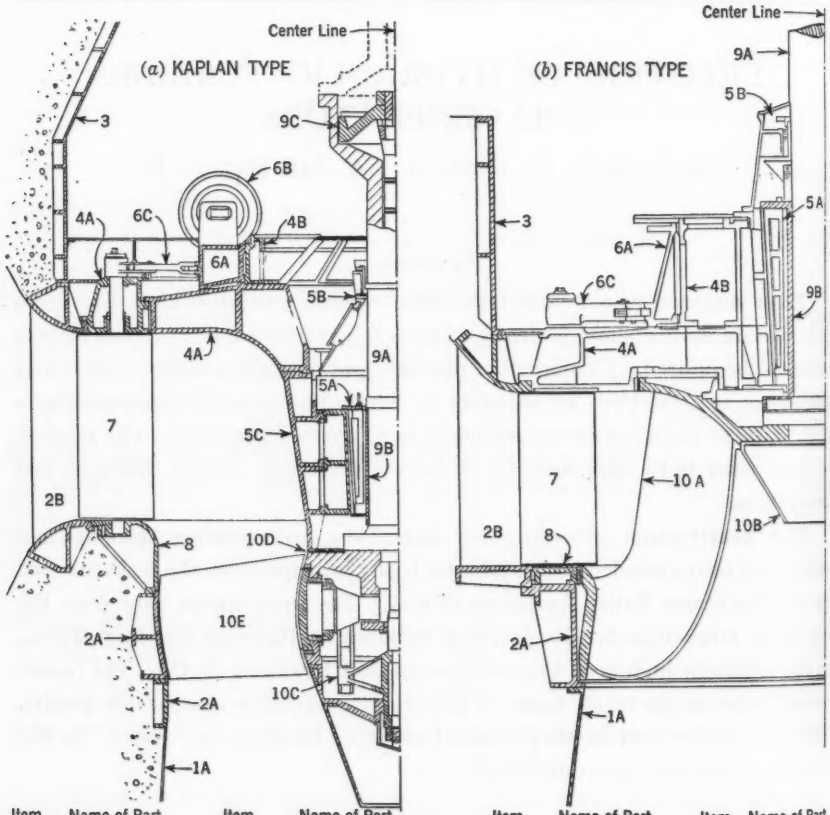
It is assumed that, just prior to placing the embedded part of the first turbine, the concrete for the power-house substructure has been poured as high as possible. In the case of a concrete scroll case, the largest part of the scroll-

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by August 1, 1944.

[1] Asst. Engr. Director, Republic of Peru, Central Hidroeléctrica del Cañón del Pato Project, Lima, Peru.

[2] "Allocation of the Tennessee Valley Authority Projects," by Theodore Parker, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 175, Fig. 1.

case floor will have been poured, including the scroll-case walls. A recess large enough for working room is left to receive the draft-tube liner. Concrete piers are erected high enough to receive the speed ring but with sufficient clearance for placing screw jacks and steel shims under it.



Item	Name of Part	Item	Name of Part	Item	Name of Part	Item	Name of Part
1A	Draft Tube Liner	5A	Guide Bearing	7	Guide Vanes	10B	Runner Tip
2A	Discharge Ring	5B	Stuffing Box	8	Guide Vane Ring	10C	Runner Blade Mechanism
2B	Speed Ring	5C	Head Cover Barrel	9A	Main Shaft		
3	Pit Liner	6A	Shifting Ring	9B	Shaft Sleeves	10D	Runner Hub and Cap
4A	Cover Plate	6B	Operating Cylinder and Rods	9C	Reservomotor Piston		
4B	Cover Plate Extension	6C	Levers and Links	10A	Runner	10E	Runner Blades

FIG. 1.—SECTIONS OF TYPICAL TURBINE TYPES

The draft-tube liner is then set in place, centered, and alined approximately to grade. It is anchored securely to U-bolts placed in the concrete. The speed ring is next assembled, either in place or on an assembly deck, depending on handling facilities and space requirements. Great care is necessary to level the ring as closely as possible. Strong supporting blocks, preferably cut from H-sections, are used in addition to screw jacks. It is not necessary to spend much time centering the ring on the theoretical power-house center lines or to

bring it exactly to the grade called for on the plans (see Stage 6). The entire procedure for the unit assembly is dependent on the final elevation and center lines of the speed ring. The lower and upper discharge rings are assembled in place and centered as closely as possible to the speed-ring center. The ultimate runner clearances will depend on how accurately this phase of the work is done. At this point all bolted contact joints have been coated with red lead and linseed oil.

After assembling the rings, the draft-tube liner is fastened to the lower discharge ring. The method of fastening varies with different manufacturers

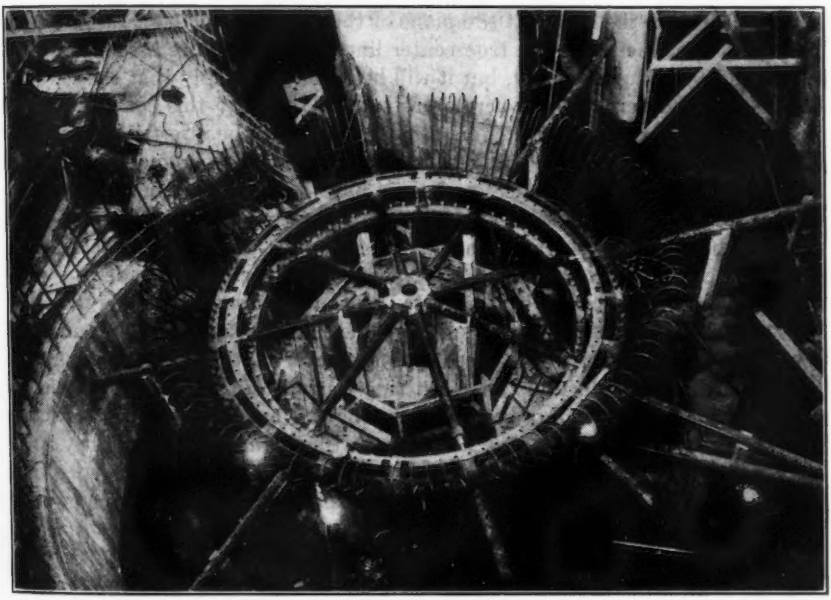


FIG. 2.—SPEED-RING ASSEMBLY SHOWING METHOD OF BRACING

and installations, but most modern draft-tube liners are welded to the discharge rings. Careful "stitch" welding in light layers and peening avoid the danger of distorting the parent metal.

Double extra strong 6-in. pipe, or its equivalent, is best to brace the speed ring rigidly, inside and outside (see Fig. 2). It is false economy to use light struts since good substantial center struts can be made by most job forces. One "spider" will suffice for many installations by merely splicing it to the arms, or by cutting, to fit various speed rings of different diameters. The jacking bolts at the ends of the struts should be about 2 in. in diameter and should be tack-welded to the ring. The external struts should be placed opposite those on the inside of the speed ring to avoid distortion. They should be supported by shores at close centers to preclude the possibility of sag. The ring should be "trued up" for roundness by jacking with the struts, and then it should be leveled.

In leveling, it is necessary to erect a separate tower for the instrument. Whether a level or a transit is used depends on the engineer's preference. The sighting target can be made up of cross hairs mounted on a micrometer head which in turn is mounted on a heavy steel base. The speed ring, the discharge rings, and the draft-tube liner are now ready for concrete and grout (see Stage 2).

The lower and upper pit liners are assembled on to the speed ring after the ring has been completely grouted. Reasonable care is needed in alining and centering the pit liner, but this precaution should not be carried to extremes. The principal thing is to center the operating cylinder bases, as closely as possible, on the center line of the unit. Of primary importance to designers and manufacturers is the fact that the plane of the operating cylinder bases cannot be held perpendicular to the true center lines of the cylinder assembly. The bases may be alined perfectly, but it will be found invariably that they are out of line after the scroll-case concrete has set. To remedy this condition, some manufacturers provide a "dutchman" or filler piece to be inserted between the operating cylinder and the base on the pit liner. Measurements are taken between the operating cylinders and their bases after final alinement and the filler pieces are machined so as to take up any discrepancy. It is recommended that the bases be made thick enough to allow for facing in the field. This operation can be performed with a job-constructed facing rig made of a slow-speed air motor to which a heavy slotted plate with a cutting tool is mounted.

A poor practice among some designers and manufacturers is to require that the pit liner be welded to the speed ring for watertightness. There is always a good chance of distorting the speed ring during the welding operation. A watertight joint can be obtained by providing a V cut in the speed-ring flange or pit-liner base (possibly both), and by inserting a rubber filler slightly larger than the V cut. Care must also be exercised to avoid distorting the speed ring in welding the various membering joints in the pit liner. Welds should be made in thin layers, and peened.

Stage 2. Grouting of Speed Ring

The speed ring on a hydraulic unit is generally designed to support a large part of the superimposed scroll-case roof, the entire generator, including rotor, the water-wheel runner, and the hydraulic thrust. It is highly essential, therefore, that a good grouting job be obtained under the speed ring, to avoid the possibility of an open joint or seam.

The concrete around the lower and upper discharge rings should be poured in relatively short lifts to within about 6 in. of the speed ring, the actual distance depending on the design of the speed ring and on job conditions. Then the concrete is allowed to stand for seven or eight days, being cooled with a water spray to shrink it as much as possible.

Procedures and proportions for mixing grout in turbine installations probably vary more with different construction organizations than any of the other operations. In an effort to devise a consistent standard, a series of grouting tests was conducted at Wilson Dam during the fall and winter of 1941. As a result of these studies the writer recommends that grout headers, about 4 or 5 ft high and spaced at 3-ft or 4-ft centers, be inserted in the speed ring. He

prefers them to be 1½ in., or larger, in diameter, with funnels to facilitate pouring grout. Small vent holes, 0.5 in. to 0.75 in., should be provided in the speed ring, spaced in a staggered manner, one being placed in each ribbed corner. Pouring should begin simultaneously at several points on the speed ring and the grout should be worked uniformly away from each header. For one to two hours, or until all signs of free water and laitance have disappeared, the grout should be rodded thoroughly. When there is evidence that pure grout is escaping from any vent hole, this hole is plugged. This procedure is followed on all vent holes around the speed ring, and all vents are reopened at about 10-min intervals and checked for free water.

The following specifications are recommended in grouting turbine speed rings:

1. Water-cement ratios are to be as low as possible, but not lower than 0.50 in confined areas;
2. Coarse aggregates are to be added to the grout as working conditions will permit; and
3. The grout is to be rodded continually for at least an hour, or until all of the free water and laitance is expelled.

Stage 3. Pouring Concrete

Designers will lay out the pours around a unit in equal segments, say 4 to 6 sections, depending on the physical size of the turbine. Alternate segments will then be poured so as to equalize the radial thrust on the speed ring and pit liner. As a general rule, a seven-day interval (or more) is necessary between horizontal lifts and a three-day interval between vertical lifts. This will allow for more rapid cooling of the individual pours, thus minimizing the shrinkage effect on the unit. Alternate lifts are poured simultaneously and at slow rates not to exceed 1 ft an hour. In no case should mechanical vibrators be allowed closer than 10 ft from the unit as there is always danger of jarring loose a strut or stay, thus causing possible misalinement or distortion of the speed ring.

Stage 4. Erection of Turbine Internal Parts Including Stationary and Rotating Elements

The starting point of all assembly of the internal parts of the turbine is the guide-vane ring. In some installations, this is fixed at the time the speed ring and upper discharge ring are installed and is an integral part of these members (see point 8, Fig. 1(*a*)). In other installations, the guide-vane ring is a separate member; it is necessary that the ring be centered as closely as possible to the discharge ring and fixed in its final position by dowels.

The outer cover plate is assembled in place and bolted loosely to the speed ring. A steel mandrel, having the same diameter as the guide-vane trunnions, is inserted through the upper guide-vane bushing in the outer cover plate and into the lower guide-vane bushing in the guide-vane ring. This check is made in each set of bushings. It may be necessary to shift the outer cover plate during this operation in order to aline the upper and lower bushings properly. In some instances, it may be necessary to scrape the bushings to prevent binding

of the guide-vane stems. When the bushings are properly alined, the outer cover plate is bolted rigidly to the speed ring and a sufficient number of dowel holes are drilled to fix the position of the outer cover plate with respect to the speed ring.

The outer cover plate is then raised as a unit and blocked up on timber cribbing; the guide vanes are next installed in accordance with factory match marks; the outer cover plate is lowered in place, bolted and doweled to the speed ring; and each of the guide vanes is centered vertically so that the clearance is the same between the guide vane and the guide-vane ring and outer cover plate.

The inner head cover (the lower point 4A, Fig. 1(a)) is bolted to the head cover barrel (point 5C, Fig. 1(a)). (Dowel holes joining these pieces usually are drilled in the factory.) These two pieces should be installed as a unit. The head cover barrel is centered with the discharge ring and the inner cover plate is then doweled to the outer cover, using undersize dowels. The cover-plate extension (point 4B, Fig. 1(*a*)) and shifting ring (point 6A, Fig. 1) are installed in the order named. During this operation, the runner is generally being assembled on an erection pedestal in another part of the building. The operating cylinders (point 6B, Fig. 1(*a*)) are assembled to their bases on the pit liner. Great care must be exercised in this operation to aline the cylinders concentrically with the shifting ring. The rods are generally alined with the shifting ring at the mid-stroke position in order to equalize the radial throw-out between the fully open and closed position. As mentioned, the pit liner will be distorted somewhat due to the shrinkage of the concrete. It would be a fortunate coincidence if the operating cylinder base stayed in alinement. In most cases it is necessary to insert a machined filler plate between the operating cylinder and the pit-liner support. A better method is to have the manufacturer weld a base plate of sufficient thickness to the pit liner to allow for machining on the job. Accurate measurements, for alinement requirements, can be taken during the assembly of the operating cylinders and the base plate can be milled with a job-facing tool. The operating cylinders and shifting ring are to be removed.

The runner and shaft assembly presents no serious difficulties. The clearances between the runner blades (point 10E, Fig. 1(*a*)) must be equalized, if necessary by inserting shims in the operating links (point 10C, Fig. 1(*a*)). The packing around the runner hub is inserted carefully and tightened with the gland ring. The hub is tested with air pressure to insure the tightness of the packing. The inner cover plate (lower point 4A, Fig. 1(*a*)) is removed from the turbine pit and mounted on the runner assembly and bolted to the head cover barrel (point 5C, Fig. 1(*a*)). The stuffing-box housing (point 5B, Fig. 1(*a*)) is now mounted around the shaft. This provides a temporary support for the runner and shaft assembly. The entire assembly is next placed in the turbine pit.

The inner head cover is bolted and doweled rigidly to the outer head cover. The runner and shaft are centered as closely as possible in the upper discharge ring (point 2A, Fig. 1(*a*)) and the head cover barrel, respectively. The shaft is then plumbed as accurately as possible. It is advisable to place jacking struts between the shaft and some convenient point on the inner head cover to

hold the shaft in vertical alinement during the generator assembly. Fixed tram points are placed in the inner head cover just above the lower guide bearing. It will be necessary to use these points after the turbine shaft is coupled to the generator shaft. The guide-bearing housing and shoes should be inserted and centered about the center of the shaft throw-out. Generally, running clearances of from 0.004 in. to 0.006 in. are used in the final adjustment. It is advisable to use long copper shims between the turbine guide-bearing shoes and the shaft. The shoes are jacked firmly against these shims and left in this position until the generator shaft is alined to the turbine. In this connection, "wedge-type" shoes are most practicable—that is, shoes with tapered back seats—because, as wear occurs, they can be readjusted easily to maintain the proper running clearances.

Stage 5. Erection of Generator-Mechanical Features

Only the mechanical features of generator erection which tie in to the turbine are discussed in this paper. Generator erection in detail has been covered capably elsewhere.[3,4]

Great care should be exercised in establishing the correct grades for the main bracket and stator sole plates (6F). The turbine shaft generally will be resting on a temporary seat from one fourth to three fourths of an inch below its final grade before being coupled to the generator. This clearance, plus an additional allowance of from 0.030 to 0.050 in. for main bearing-bracket deflection (depending on the bracket span and loading) should be taken into account when computing the final elevation of the sole plates.

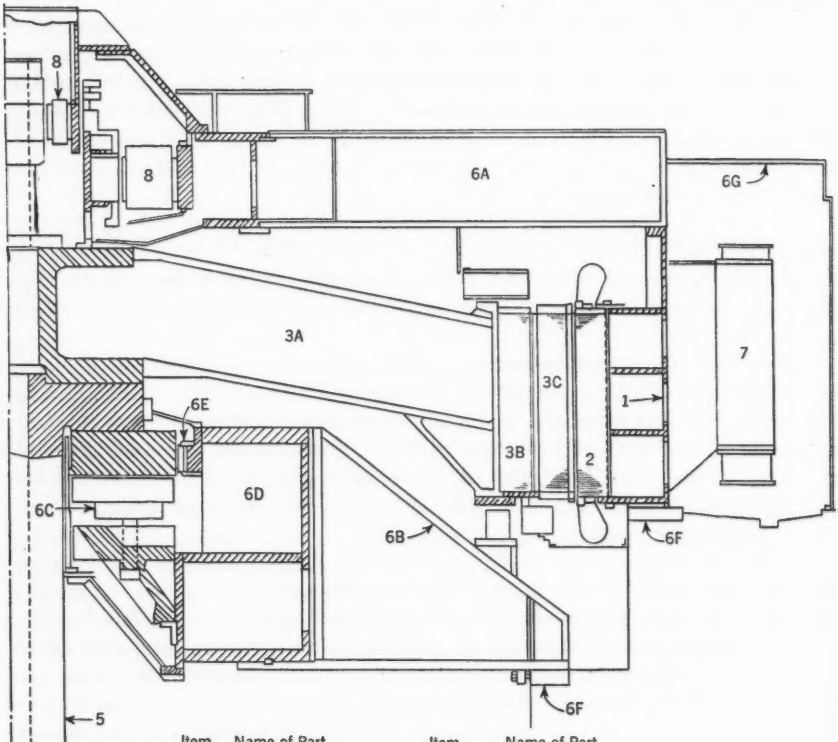
It is highly desirable to center and aline the generator shaft and bearing bracket to the turbine shaft before final centering and alinement of the stator. The generator shaft is centered on the thrust bearing before being placed in the pit. The thrust-bearing shoes are all set to initial adjustment before the rotor is placed. At this point both the main bracket and stator sole plates are grouted. It has been found that grout with a water-cement ratio of 0.40 is best suited for this purpose. This grout is dry enough so that it can be placed under the bearing plates without the use of forms and does not shrink on setting up.

Some generator manufacturers use a radial type bracket, whereas others use through girders with side arm stiffeners. In the latter case, it is recommended that the side arms be not fastened and doweled to the sole plates until the unit has been loaded to about three quarters of its rated capacity. The added deflection caused by the hydraulic thrust will tend to close-in the guide-bearing shoes (point 6E, Fig. 3) and thus cause excessive heating if the side arms are fastened down initially.

After the bearing-bracket sole plates are grouted, the remainder of the generator erection can be completed. The generator shaft is centered accurately and plumbed to the turbine shaft. Four heavy plumb bobs suspended on piano wires at the quarter points can best be used for this purpose. Using temporary coupling bolts, the turbine shaft and runner are brought up slowly

[3] "Order Out of Chaos," by H. A. Wallace, *Allis-Chalmers Electrical Review*, September, 1941.
[4] "Slugged Arc," by H. A. Wallace, *ibid.*, June, 1942.

to the generator shaft. It is customary practice to make the permanent coupling bolts 1 to 2 mills oversize, which necessitates freezing the bolts in dry ice or forcing them in place with hydraulic jacks. Either method is satisfactory and the choice depends on job conditions. The bolts are pre-stressed uniformly



Item	Name of Part	Item	Name of Part
1	Stator Frame	6B	Lower Bracket
2	Stator Punchings	6C	Thrust Bearing
3A	Rotor Spider	6D	Bearing Oil Cooler
3B	Rotor Rim	6E	Guide Bearing
3C	Rotor Poles	6F	Soleplates
4	Stator Windings	6G	Housing
5	Main Shaft	7	Air Coolers
6A	Upper Bracket	8	Exciters

FIG. 3.—SECTION THROUGH TYPICAL GENERATOR

and to such an extent that the total tension developed will be in excess of the combined weight of the turbine, turbine shaft, and the maximum hydraulic thrust, and, in the case of the Kaplan types, the maximum internal pressures. This will assure that the turbine and generator shaft coupling faces are in positive contact under all conditions. In the old days, the general order used to be to "slug her up till she rings and then take another turn." In some instances, this resulted in a permanent deformation in the coupling flanges with resultant

misalinement of shafts. The customary stretch in bolts is from 0.006 to 0.012 mill, depending on the number, size, and length of bolts, and on the total suspended load.

The customary procedure is to have the turbine shaft shipped to the generator manufacturer for final fitting up to, and alining with, the generator shaft —a needless expenditure of time and money. With present precision machining methods, the chance of misalining the coupling flanges is negligible. The holes for the coupling bolts can be subdrilled in the shop $\frac{1}{8}$ in. undersize and line reamed in the field after the two shafts are drawn together. One generator manufacturer uses this method in fitting his rotor to the shaft in the field. For a cutting unit, he uses an automobile cylinder honing machine fitted up with a cutting tool.

After the turbine shaft is coupled permanently to the generator shaft, the entire unit is free to swing on the thrust bearing. The thrust-bearing shoes are set up as uniformly as possible. They are generally adjusted by either the "feel" method or the "slugged arc" [4] method. The first requires a highly skilled mechanic and the second is much more accurate. By "feel" each shoe is adjusted progressively around the thrust bearing. The amount of tension on each adjusting screw is determined by the mechanic by observation and "feel" of the wrench. At best, this is a guess on the operator's part. Basically, the "slugged arc" method consists of measuring an arc at a known radius of a hand-calibrating wrench. Each of the adjusting screws under the thrust shoes is loosened completely, progressively and one at a time. The screws are tightened by a hand-calibrating wrench, care being exercised to obtain the same tension on each screw. The arc is measured by a graduated scale and the values are plotted. The loose shoes are tightened and the tight shoes are loosened by amounts depending on the plotted values. The "slugged arc" measurements are repeated and the values again plotted. Theoretically, they should all fall on the same horizontal line. An experienced operator generally can make a uniform adjustment on the second trial.

The following method of adjusting thrust-bearing shoes was evolved on the job by G. L. Teague and has been used successfully on two large machines (see Fig. 4). The combined assembly consisting of rotor, generator and turbine shafts, and runner is plumbed as accurately as possible. The initial thrust of each individual thrust shoe is immaterial. Two indicators are placed at the lower guide bearing at the quarter points (see Fig. 4(*c*)).

Compression is applied on the thrust shoes progressively, beginning with shoe No. 1 and continuing to shoe No. 8 (Fig. 4(*b*)). If either indicator shows the least movement during the adjustment on any individual shoe, the setting is stopped and the process is repeated on the next shoe in numerical order. Obviously the setting up of shoe No. 1 will not produce any movement in the west-east direction, but will produce motion in the north-south direction. The adjustment of shoe No. 2 will produce a movement which will show up on both indicators in Fig. 4(*c*). The procedure is repeated until the slightest adjustment on any individual shoe produces a movement on either indicator. This simple method will work with any number of thrust shoes.

It is absolutely essential that, whatever method is used, each thrust shoe be adjusted to as uniform a loading as possible. The successful running of the unit will depend to a very high degree on this adjustment.

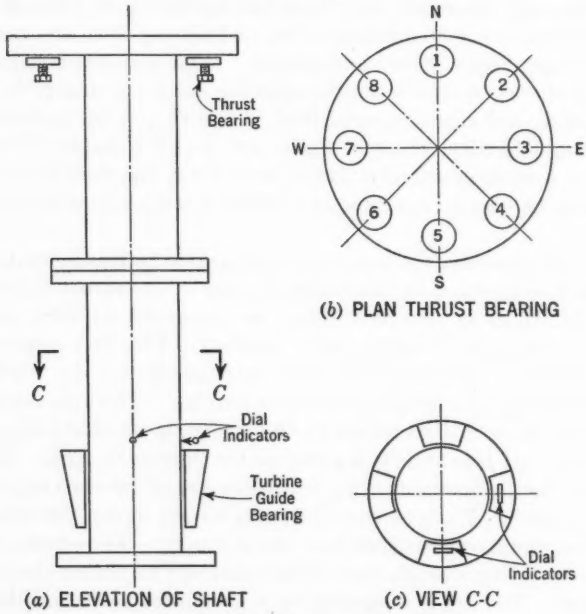


FIG. 4.—THRUST-BEARING ADJUSTMENT

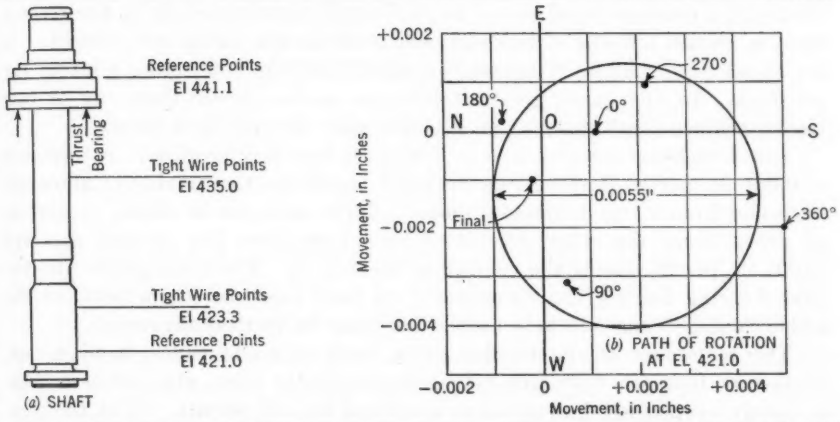
In recent years, it has been customary to make a "rotation" check on the combined assembled unit. This check consists in rotating the unit mechanically (by jacking or attaching cables from the power-house crane to the rotor spider) through four 90° turns and taking plumb readings at several points. The bearing shoes are coated with a heavy graphite and oil lubricant prior to the mechanical rotation to preclude the possibility of scarring the bearing surfaces. It has also been found advisable to disregard the readings on the initial 90° turn because the heavy lubricant is "ironed out" between the shoes and the runner place in this turn.

A typical rotation check is shown in Table 1, in which Fig. (*b*) testifies that the axis of the shaft described a circle of rotation at the lower guide bearing of 0.0055 in. The center of this circle is 2 mills south and 1¼ mills west from the theoretical center line. The initial or zero point was 1¼ mills south and zero mills east and west. The shaft was replumbed so that the final position was 1 mill west and zero mills north and south.

In a great many installations, the thrust collar is not an integral part of the shaft but is shrunk or keyed to the shaft in the field. In such cases, a rotation check would indicate whether or not the face of the thrust collar was set normal to the shaft. It is conceivable that a thrust collar could be set a few mills from a normal position and the setting of the thrust shoes equalized in this position. Plumb readings would indicate that the shaft was vertical and

centralized in the turbine guide bearing. However, in actual operation the shaft would describe a cone; that is, it would wobble within the turbine guide bearing with an amplitude depending on the ratio of the bearing diameter to the combined length of the turbine and generator shafts and the amount the thrust

TABLE 1.—OBSERVATIONS FOR A TYPICAL ROTATION CHECK



Direction	Reference		Micrometer Readings for Reference Points						Centering[a]
	Tight wires	Reference points	0°	90°	180°	270°	360°	Final	
(a) Elevation 439.9 (Tight Wire Elevation 435.0)									
N	0.495	0.5315	0.529	0.530	0.5315	0.533	0.533	0.542	0.243
S	0.504	0.5365	0.541	0.540	0.538	0.537	0.537	0.5275	0.244
E	0.492	0.6045	0.6305	0.631	0.6295	0.6305	0.6315	0.5945	0.254
W	0.499	0.576	0.5535	0.5515	0.5535	0.553	0.551	0.588	0.255
(b) Elevation 421.0 (Tight Wire Elevation 423.3)									
N	0.4765	0.7615	0.759	0.759	0.7595	0.764	0.7675	0.7705	....
S	0.4855	0.5985	0.6005	0.600	0.5995	0.5955	0.5935	0.588	....
E	0.4725	0.691	0.7145	0.719	0.7135	0.7135	0.7185	0.6795	....
W	0.480	0.769	0.744	0.740	0.745	0.745	0.7405	0.778	....

[a] These are micrometer readings for the final centering of the turbine and guide bearing (not necessarily at El. 439.9).

collar was set out of normal to the shaft. The rotation check could be omitted in the case of an integral thrust collar and generator shaft. Present-day machining methods are very precise so that there is little danger of the thrust collar being out of true with the shaft. The shaft is rotated in a lathe after machining and polishing, and careful throw-out measurements are made with dial indicators.

The slight added cost of an integral thrust collar and shaft is warranted over that of keyed or shrunk hub. The integral thrust hub and shaft is easier to install and the results are more certain. In some installations having shrunk hubs, several weeks were spent in alining the units.

Stage 6. Allowable Tolerances

In some installations, much unnecessary time and effort have been spent in alining or centering a unit to theoretical elevations and center lines. Deciding the necessary degree of accuracy requires a certain amount of "horse sense." Obviously, in erecting a high-head unit with a steel scroll case, which in turn connects to a steel penstock, closer tolerances are adhered to than in a low-head unit with a concrete scroll case. In steel scroll cases, especially in the riveted type, a certain amount of creepage will occur as the plates are assembled to the speed ring, making it necessary to field drill the connecting joint to the penstock. In such cases, greater tolerances can be allowed than, say, in cast steel machined scroll cases where it is necessary to meet fixed points.

Speed rings are set and held as level as is humanly possible. A tolerance of 0.001 in. per ft of speed-ring diameter should be the maximum allowable deviation from a true horizontal plane. There are cases in which, in spite of all precautions, the rings get out of level more than the amount specified (0.001 in. per ft) during the concreting operations. The cover plates and the guide-bearing housing can be adjusted, in such cases, to compensate for the added "outage" and still hold the turbine shaft in vertical alinement.

The remainder of the turbine parts, such as guide vanes, head covers, guide-vane linkages, operating cylinders, distributor rings, etc., are assembled as closely to specified dimensions as machined fits will permit. With the foregoing suggestions in mind, it is possible to summarize the principal erection tolerances as shown in Table 2.

TABLE 2.—Summary of Principal Erection Tolerances, in Inches

For:	Speed Rings		Operating cylinders	Generator		Shaft alinement
	Concrete scroll cases	Spiral, steel scroll cases		Stator	Shaft	
Elevation	±0.060	±0.030	±0.010	±0.030[b]	....	....
Centering	±0.090	±0.060	±0.060[a]	....	....	....
Concentricity	....	....	....	±0.030	....	....
Turbine shaft	....	....	....	....	±0.010[c]	....
Plumb	....	....	....	....	....	0.001[d]

[a] For centering horizontally. [b] For elevation, with respect to rotor. [c] Base coupling with respect to turbine shaft. [d] 0.001 in. per 5-ft height of shaft, from the true vertical.

Acknowledgments

Engineering work for the TVA was under the general supervision of Theodore B. Parker, M. Am. Soc. C. E., chief engineer; George R. Rich, M. Am. Soc. C. E., chief design engineer; A. L. Pauls, M. Am. Soc. C. E., chief construction engineer; and H. J. Petersen, head of the Mechanical Design Section.

The writer wishes to express his sincere thanks and appreciation to the many turbine mechanics and erectors employed by the Authority, and especially to J. S. Kuzawa, formerly master mechanic, for their many helpful suggestions and instructions in the art of assembling turbines; and to Sam McCluer, associate materials engineer, for conducting the grouting tests.

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PAPERS

FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS

By H. B. Kinnison,[1] M. Am. Soc. C. E., and B. R. Colby,[2] Esq.

Synopsis

The determination of the magnitude and frequency of expected floods has been the subject of constant study by the engineering profession. The systematic collection of stream-flow records in every section of the United States has contributed immeasurably to a knowledge of the subject. The long-time records of precipitation assist in extending the frequency studies beyond the range of stream-flow records. Contributing to this knowledge also have been the disastrous floods since 1927, which in many instances exceeded the maximum expected, based on previous records.

In this paper, flood formulas are developed from stream-flow records of Massachusetts streams, for which fairly complete data are available. The formulas define the magnitude of flood peaks having probable recurrence intervals of 15, 100, and 1,000 years, and the maximum possible flood.

Volumes of runoff per unit area for the different classes of floods except the maximum were computed for streams having the longest stream-flow records, from frequency curves of observed floods and of observed precipitation. The peak discharge of each type of flood was computed for these streams by means of the unit hydrograph. These discharges were then correlated with known physical characteristics of these streams, and general equations were prepared which are applicable to both gaged and ungaged areas.

These formulas are based upon data for drainage basins in and adjacent to Massachusetts and therefore are conceived as applicable only to basins of that state or basins having similar physical and meteorological characteristics. The formulas might assume a different form in the case of drainage basins in other sections of the United States. The primary purpose of the paper is to present

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by August 1, 1944.

[1] Dist. Engr., U. S. Geological Survey, U. S. Dept. of Interior, Boston, Mass.

[2] Associate Engr., U. S. Geological Survey, U. S. Dept. of Interior, Boston, Mass.

an extensive analysis of flood experience which engineers may embody in their knowledge of floods and utilize with some benefit in problems involving flood magnitudes and frequencies.

INTRODUCTION

Numerous methods of predicting future flood flows have been developed and described in engineering publications. Prior to 1900 these methods were necessarily rather crude, because authentic records of stream flow were scanty and less accurate than later records. Practically all such studies were based on observations of flow in small areas or on single streams, and were usually summarized as formulas giving the expected rate of flow as a function of the drainage area, although some investigators added other variables. These early formulas were usually developed for maximum floods, the meager base data precluding any except the roughest qualitative consideration of frequency.

In 1913, the late Weston E. Fuller,[3] M. Am. Soc. C. E., published the first comprehensive study of statistical methods applied to floods in the United States. He developed a formula introducing the concept of a magnitude-frequency relationship, in this respect following a line of reasoning that was introduced into studies of river flow about the same time by the late Allen Hazen, M. Am. Soc. C. E. Mr. Hazen's application of statistical methods to engineering studies and Mr. Fuller's adaptation of these methods to flood prediction have had an important influence on engineering developments since that time.

Shortly after the Civil War, the late Maj. E. T. D. Myers, M. Am. Soc. C. E., developed a formula for computing the cross-sectional area of the waterway required to pass a limiting flood. The Modified Myers formula[4]—

$$Q = C_m\sqrt{M} \qquad (1)$$

—was developed from the original. In this formula, Q is the discharge in cubic feet per second, and M is the area in square miles. The base factor C_m is taken as 10,000, but for any particular area it is multiplied by a percentage which is referred to as the Myers rating. Eq. 1 has had wide application, and is discussed in recent engineering publications.[5,6] The reliability of the results obtained by it depends upon the choice of the coefficient C_m. The magnitude of the coefficient selected will be governed principally by the magnitude of the observed floods in the locality, and by the judgment and experience of the engineer.

Further studies have developed a different approach to the problem of flood prediction, by the so-called "rational" methods. These involve the relation of rainfall to the resulting runoff by an analysis of rainfall records and

[3] "Flood Flows," by Weston E. Fuller, *Transactions*, Am. Soc. C. E., Vol. LXXVII (December, 1914), pp. 564–617.

[4] "Flood Flow Characteristics," by C. S. Jarvis, *ibid.*, Vol. 89 (1926), pp. 985–1104.

[5] "Low Dams," by National Resources Committee, 1938, p. 32.

[6] "Physics of the Earth," Pt. 9, Hydrology, National Research Council, McGraw-Hill Book Co., Inc., 1942, p. 536.

corresponding hydrographs, with due consideration being given to the influence of ground water, evaporation, transpiration, shape of drainage area, and other factors. This method includes studies of the frequency of intense rainfall and its distribution over large areas. Reports on the frequency and distribution of rainfall having relation to New England have been made by the Miami Conservancy District[7] and by the U. S. Weather Bureau.[8]

As stream-flow records have become more extensive and reliable, the tabulations of extreme floods have increased in value and have made obsolete some of the earlier formulas based on very limited or deficient experience. It is inevitable, however, that the much more extensive experience now available should be analyzed with a view to expression by formulas of relationships between magnitude and frequency of floods as well as of the influence of physical characteristics of basins on the magnitude of floods. Thereby, the maximum utility may be given to the available data which, although much more plentiful than formerly, often have only a rather remote relation to the needs of a specific problem.

The "rational" method of approach has been extended in this paper. Investigations by the writers have shown that the magnitude of a flood of a particular frequency can be correlated with various physical characteristics of the drainage basins. By means of these characteristics, formulas have been developed which are applicable to drainage areas for which stream-flow records are not available or are inadequate.

The common practice in the past has been to use enveloping curves of maximum known discharge per square mile in the design of hydraulic structures. This practice ignores the element of frequency, which may have an important bearing on the risks involved. This paper undertakes to develop a basis for a more discriminative method of considering flood magnitudes in which consideration may be given both to probable frequency and to the influence of the various physical characteristics in the production of floods.

These formulas should not be expected to give exact or unquestionable results on ungaged areas. However, they do provide a convenient means of evaluating the risks involved in the design of hydraulic structures on streams in or near Massachusetts even though the drainage areas may have widely different drainage basin characteristics.

The writers are under no illusion as to the reliability of deductions that may be drawn as to frequencies of major or rare floods from a record which covers little more than 25 years. Nevertheless, it is desirable to make an attempt at such a deduction, and in this paper the results of a systematic, scientific attempt are presented. The writers conceive that the specific form of the formulas developed may not comprise a substantial or lasting contribution to engineering knowledge. However, they hope that the account of the studies and the results may be in such a form that other engineers may assimilate some worth-while ideas in their body of experience with consequent benefit to their handling of flood-frequency problems.

[7] "Storm Rainfall of Eastern United States," *Technical Reports*, Miami Conservatory Dist., Pt. 5 (revised), 1936.

[8] "A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin Above Union Village, Vt.," U. S. Weather Bureau, 1940 (unpublished).

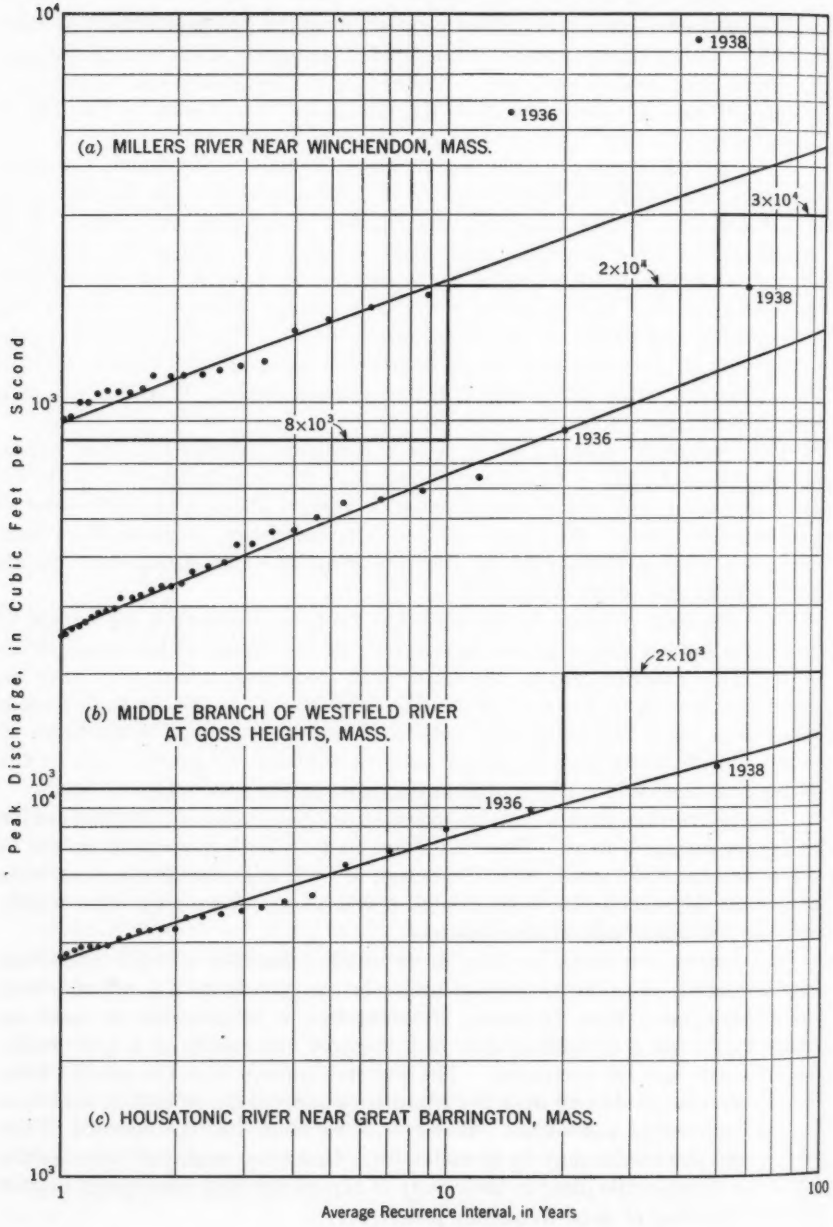


FIG. 1.—FREQUENCY OF FLOOD PEAKS

Frequency Curves of Peak Discharge from Observed Floods

The most logical method of determining the magnitude of expected floods is by a study of the frequency and magnitude of past floods. A frequency curve of instantaneous peak discharges was plotted for each gaging station having a sufficient length of record to justify its use. Fig. 1 shows frequency curves for Millers River near Winchendon, Mass., with 23 years of record, Middle Branch of Westfield River at Goss Heights, Mass., with 26 years of record; and Housatonic River near Great Barrington, Mass., with 25 years of record. In this diagram, the average recurrence interval in years is plotted against discharge. The average recurrence interval I is computed from the equation

$$I = \frac{N_y}{N_p - 0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

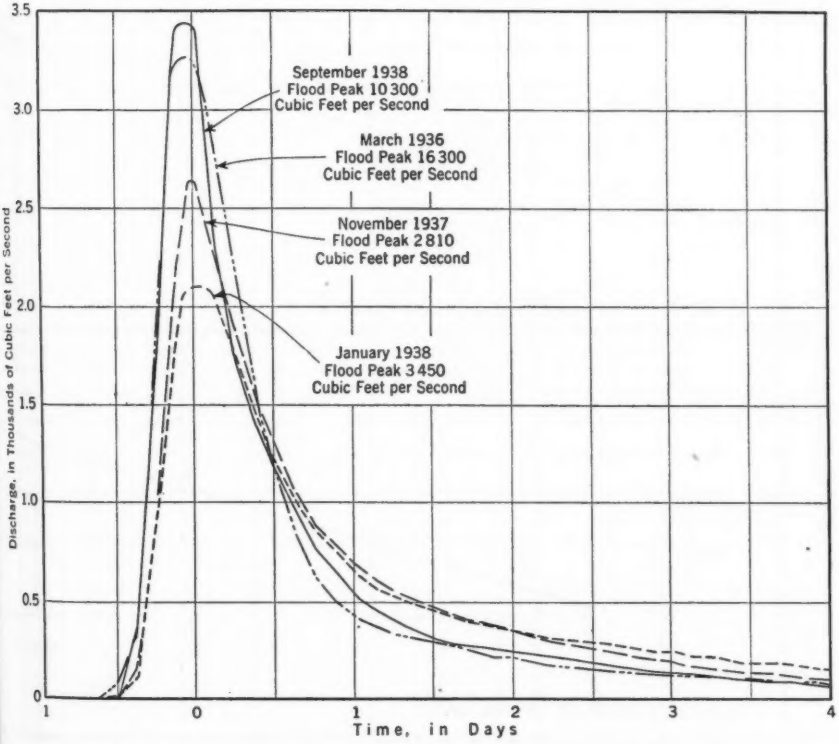


Fig. 2.—Six-Hour Unit Hydrographs Computed from Indicated Floods; North Nashua River, Near Leominster, Mass.

in which N_p is the number of the flood peak in order of size without regard to the time of its occurrence, and N_y is the number of years of record.

Widely different frequency curves can be drawn from the base data for any particular station, and extensions of the curves to recurrence intervals

of 100 and 1,000 years are not defined because of insufficient length of record. If the floods of 1936 and 1938 had not occurred during the period of record, much different curves might have been drawn. However, by allowing for extreme severity of these floods in certain areas, the magnitude of a flood having an average recurrence interval of 15 years (called a 15-yr flood) can be determined with reasonable accuracy for many streams.

Because stream-flow records are not long enough to define the frequency curves of peak discharge beyond short recurrence intervals, peak discharges at recurrence intervals of 100 and 1,000 years were computed from a study of unit hydrographs, and depths and distribution of runoff.

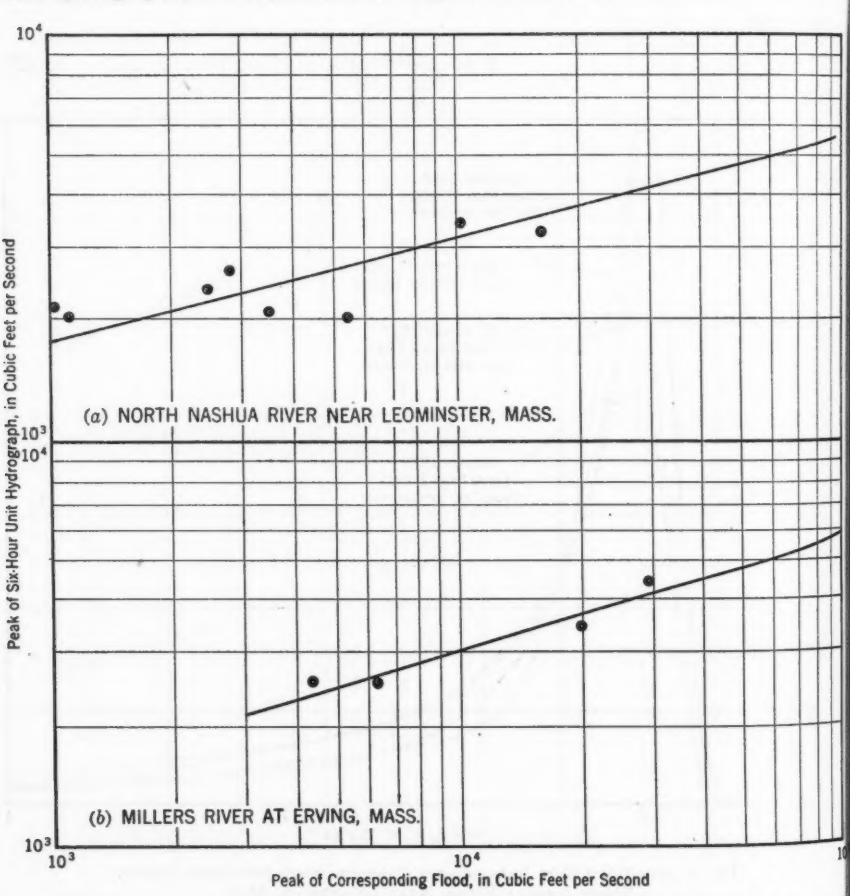

Peak of Six-Hour Unit Hydrograph, in Cubic Feet per Second
(a) NORTH NASHUA RIVER NEAR LEOMINSTER, MASS.
(b) MILLERS RIVER AT ERVING, MASS.
Peak of Corresponding Flood, in Cubic Feet per Second

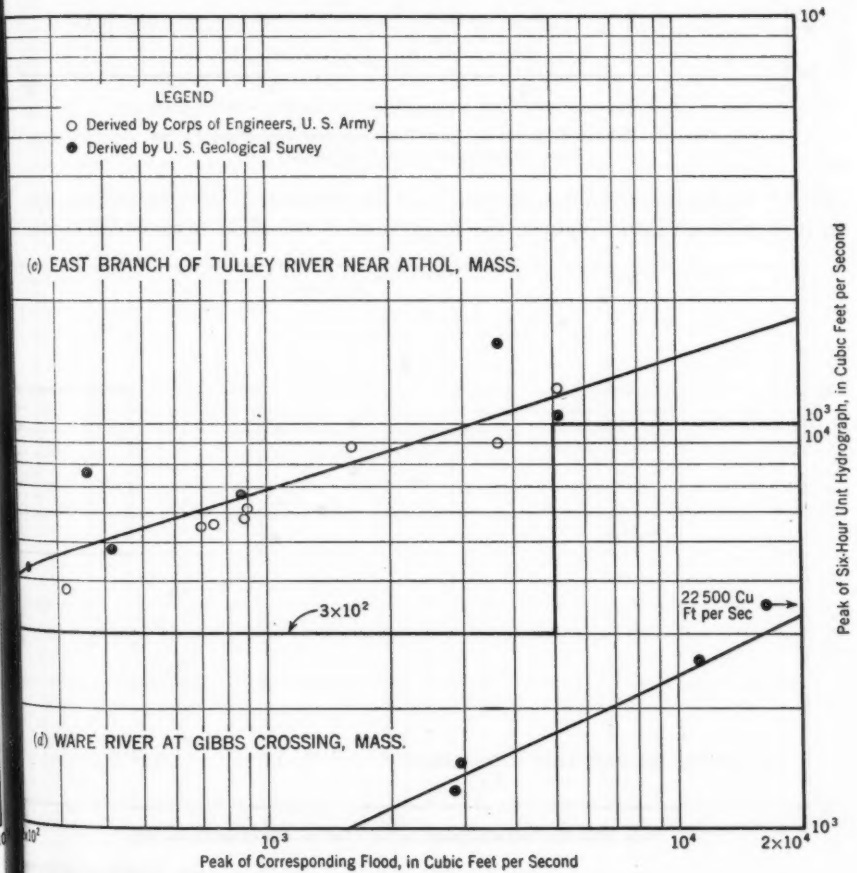
FIG. 3.—RELATION BETWEEN UNI

UNIT HYDROGRAPHS

The unit hydrograph method was used to compute the discharge hydrographs of floods from runoff data. The peak discharges were then determined by these hydrographs.

As used in this paper, the unit hydrograph is the discharge hydrograph of 1 in. of direct runoff uniformly generated over the drainage area at a uniform rate. This definition is in common use, although it differs from that given by W. G. Hoyt, M. Am. Soc. C. E., in 1936,[9] by restricting the term to uniformly generated runoff of unit depth. The term, unit hydrograph, usually is qualified by stating the duration of the runoff that generated it. For example, a 4-hr unit hydrograph is the discharge hydrograph resulting from runoff generated uniformly during the period of 4 hours at a rate of 0.25 in. per hr.

The unit hydrograph theory usually presumes that the base and shape of the unit hydrograph for a particular point on a stream will be the same for


LEGEND
Derived by Corps of Engineers, U. S. Army
Derived by U. S. Geological Survey
(c) EAST BRANCH OF TULLEY RIVER NEAR ATHOL, MASS.
3×10²
22 500 Cu Ft per Sec
(d) WARE RIVER AT GIBBS CROSSING, MASS.
10⁴
10³
10⁴
10³
10³
10⁴
2×10⁴
Peak of Six-Hour Unit Hydrograph, in Cubic Feet per Second
Peak of Corresponding Flood, in Cubic Feet per Second

...DROGRAPH PEAKS AND FLOOD PEAKS

all floods where the runoff generation is uniform over the area. Differences in the unit hydrographs derived from observed floods have been attributed

[9] "Studies of Relations of Rainfall to Run-Off in the United States," by W. G. Hoyt, *Water-Supply Paper No. 772*, U. S. Geological Survey, 1936, p. 124.

usually to the uneven distribution of runoff generation over the area. This is undoubtedly true of many unit hydrographs derived from observed floods, and such distortion must be expected. However, studies by the writers and

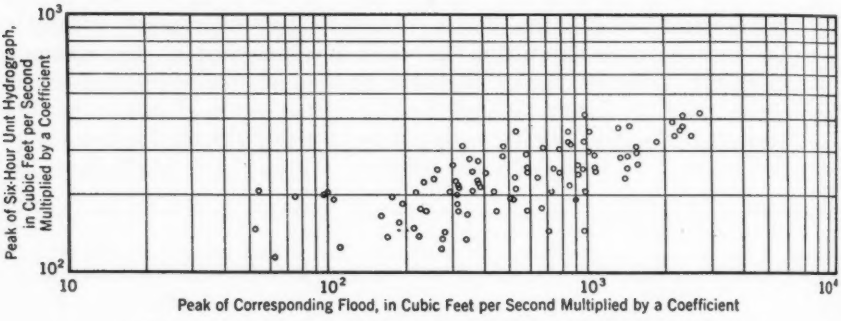

Peak of Six-Hour Unit Hydrograph, in Cubic Feet per Second Multiplied by a Coefficient
10^3
10^2
10
10^2
10^3
10^4
Peak of Corresponding Flood, in Cubic Feet per Second Multiplied by a Coefficient

FIG. 4.—RELATION BETWEEN UNIT HYDROGRAPHS AND PEAK FLOWS, BOTH MULTIPLIED BY A COEFFICIENT

others[10] have indicated a definite tendency for the peak of the unit hydrograph to increase with an increase in the magnitude of the flood from which it was

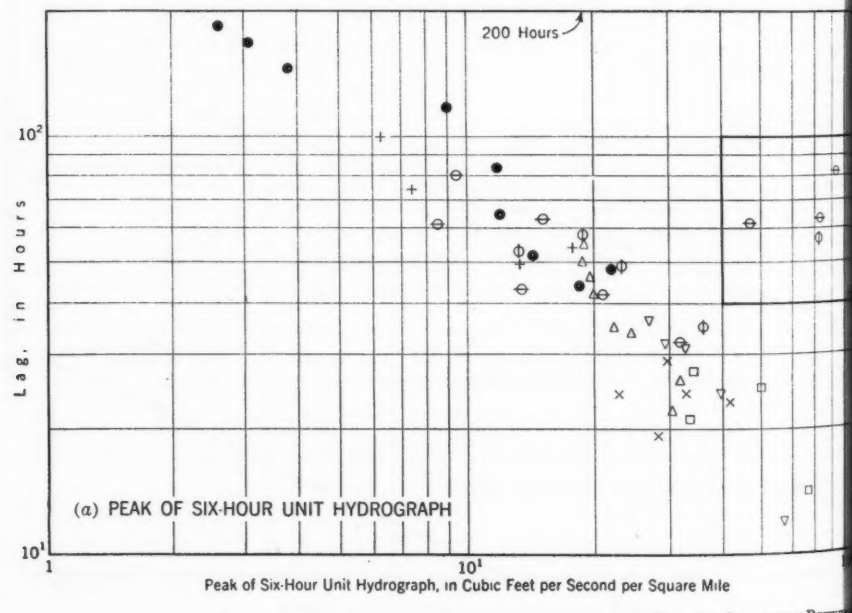


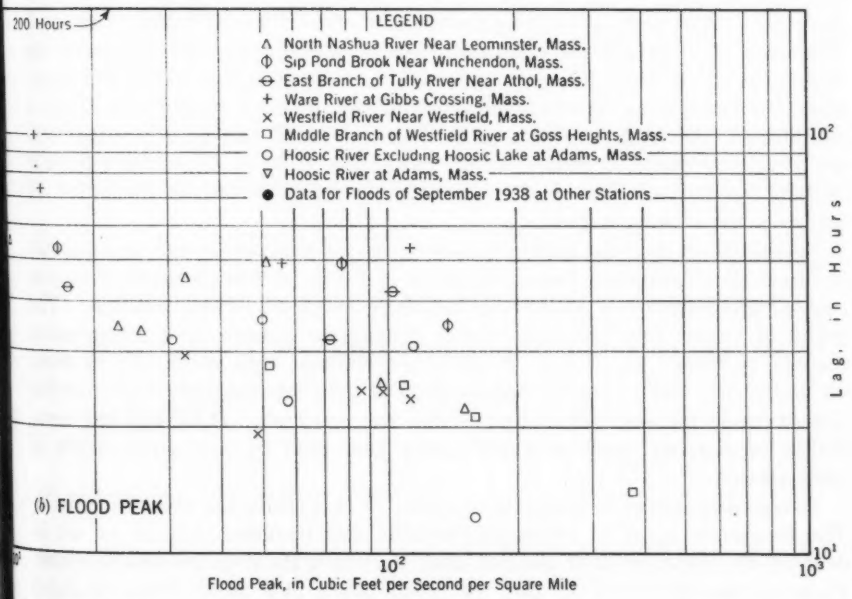
FIG. 5.—RELATION BETWE

derived. Several typical unit hydrographs for North Nashua River near Leominster, Mass., are shown in Fig. 2. These curves illustrate the difference between the peaks of the unit hydrographs derived from the higher floods and

[10] "Application of Hydrology in Flood Control," by G. A. Hathaway, *Proceedings*, Hydrology Conference, State College, Pa., 1942.

those derived from smaller floods. This tendency is shown graphically in Fig. 3, in which the peaks of several floods are plotted against the peaks of the corresponding 6-hr unit hydrographs. The general increase of unit hydrograph peaks with increase in the corresponding floods is demonstrated also in Fig. 4, in which similar data from several stations are plotted on a composite diagram. Each coordinate has been multiplied by a coefficient which would make the trend line of each station pass through a point whose coordinates are 3,000 and 400. In spite of the scattering of the points on this figure, the general upward trend is indicated clearly.

The peaks of unit hydrographs for many gaging stations in and adjacent to Massachusetts were plotted in the manner similar to Fig. 3. The trend for some stations is uncertain and it was not defined at all if the observed floods did not cover a fairly wide range of discharge. The relation, if it could be completely defined, probably would not be a straight line on logarithmic paper either for a composite graph or for an individual station. The data are insufficient to define any trend other than a straight line. However, the data were sufficient to justify the adjustment of the peaks of the unit hydrographs for the types of floods considered in this study. Extensions of the relation curves for extremely large floods are inexact, but certainly are more accurate


200 Hours
LEGEND
North Nashua River Near Leominster, Mass.
Sip Pond Brook Near Winchendon, Mass.
East Branch of Tully River Near Athol, Mass.
Ware River at Gibbs Crossing, Mass.
Westfield River Near Westfield, Mass.
Middle Branch of Westfield River at Goss Heights, Mass.
Hoosic River Excluding Hoosic Lake at Adams, Mass.
Hoosic River at Adams, Mass.
Data for Floods of September 1938 at Other Stations
(b) FLOOD PEAK
10^2
10^1
10^3
Lag, in Hours
Flood Peak, in Cubic Feet per Second per Square Mile

than results obtained by assuming no increase in unit hydrograph peaks with increase in the size of the flood.

W. B. Langbein,[11] Assoc. M. Am. Soc. C. E., has shown that the peak of

[11] "Channel-Storage and Unit-Hydrograph Studies," by W. B. Langbein, *Transactions*, Am. Geophysical Union, Pt. 2, 1940, pp. 620–627.

unit hydrographs, as expressed by the S-curve, increased with a decrease in the time by which the center of mass of discharge lagged behind the center of mass of runoff generation. "Runoff generation" as used in this paper is defined as the origination of direct runoff, and "direct runoff" is that part of the precipitation which flows more or less over the land surface. The lag time was computed from the recession curve by the method suggested by Mr. Langbein for different floods at each of several gaging stations, and for the flood of September, 1938, at several other stations. Fig. 5(*a*) shows the relation between the lag time and the peak of the unit hydrograph. In Fig. 5(*b*) the lag time, in hours, for different floods at seven gaging stations is plotted against the peak discharge of these floods. The tendency for this lag time to decrease with an increase in size of flood peak, in cubic feet per second per square mile, is shown both for the different floods at a single station and for floods at different stations. Thus, Fig. 5 confirms the increase in the peak discharge of the unit hydrograph with an increase in the peak discharge of the corresponding flood.

Frequency Curves of Depth of Runoff

In order to use the frequency curves of precipitation in a study of the frequency of peak discharges, the use of frequency curves of depth of runoff is desirable. Although a relationship exists between precipitation and peak discharge, a much closer relationship exists between precipitation and the volume of runoff which is usually expressed as a depth in inches over the basin. Once this correlation between precipitation and depth of consequent runoff is established, the frequency of a given depth of runoff can be computed from the frequency curve of precipitation. When the depth of runoff for a flood of certain frequency is known, the discharge hydrograph can be computed by means of the unit hydrograph.

To establish the relationship between precipitation and runoff, the amount of direct runoff resulting from each storm of 1.5 in. or more precipitation over each of three areas was plotted against the corresponding precipitation. The depth of runoff was determined from stream-flow records, and precipitation records published by the U. S. Weather Bureau were used. From these plotted points and a general consideration of the probable limits and distribution of runoff factors, a probability table was prepared. It showed the probability of a given depth of runoff being generated by any given depth of precipitation.

Frequency curves of precipitation used in this study are shown in Fig. 6. The frequency curve of 24-hr precipitation was included only as an aid in defining the curve of 72-hr precipitation, for which the data are less complete. These curves are based on data for precipitation at a point, from "Rainfall Intensity-Frequency Data"[12] and "Storm Rainfall of Eastern United States."[7] The upper ends of the frequency curves are based on the maximum precipitation as shown in the Miami Conservancy Report of 1936;[13] the maximum pos-

[12] "Rainfall Intensity-Frequency Data," by David L. Yarnell, *Miscellaneous Publication No. 204*, U. S. D. A., 1935.

[13] "Storm Rainfall of Eastern United States," *Technical Reports*, Miami Conservancy Dist., Pt. 5 (revised), 1936, Figs. 38 and 40.

sible precipitation on an area of 100 sq miles as given in the Ompompanoosuc Report of the Weather Bureau in 1940;[14] and the maximum point rainfall of the northeastern United States from unofficial records as indicated by an extension of a curve in the Ompompanoosuc Report.[15] These maximum amounts of precipitation are plotted at arbitrary recurrence intervals of 10,000 or 100,000 years to provide some basis for an extension of the frequency curves.

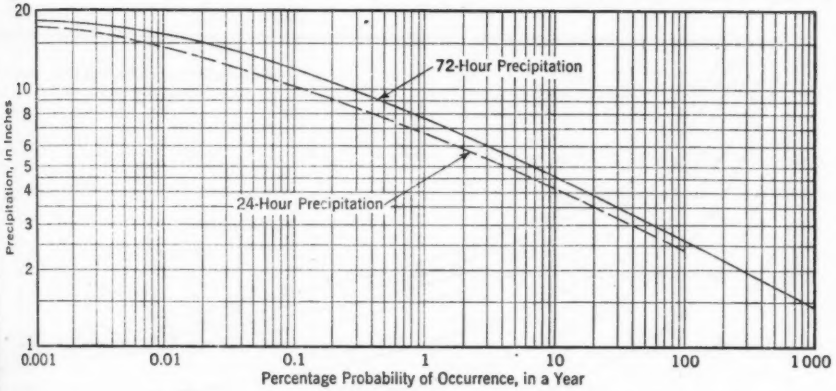


FIG. 6.—FREQUENCY OF SEVENTY-TWO HOUR PRECIPITATION FOR DRAINAGE BASINS IN MASSACHUSETTS

The frequency curve of precipitation in 72 hours was assumed to be applicable to drainage basins in Massachusetts, although certain assumptions had to be made in drawing it. The upper end of the curve was drawn to represent frequency of precipitation on drainage areas of 100 or 200 sq miles. At

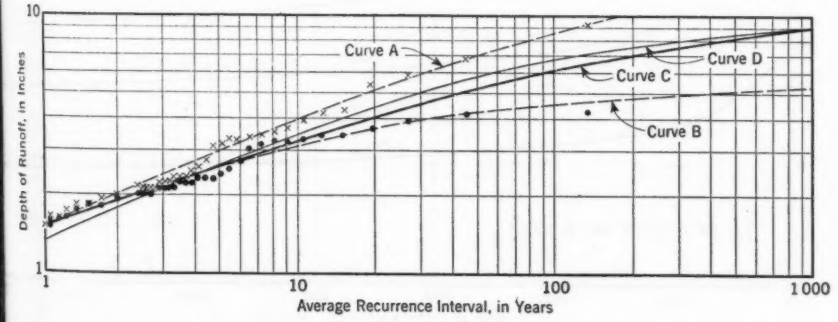


FIG. 7.—FREQUENCY CURVES OF DEPTH OF RUNOFF; COMBINED RECORDS FOR TAUNTON RIVER, PRIEST BROOK, AND THE MIDDLE BRANCH OF WESTFIELD RIVER

average recurrence intervals of 100 years or less, the curve is based more or less directly on observations of precipitation at a point, so that it does not apply exactly to precipitation over the actual drainage area. The difference

[14] "A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin Above Union Village, Vt.," U. S. Weather Bureau, 1940, Table VI (unpublished).

[15] *Ibid.*, Fig. 24.

between point and areal precipitation probably is not great on drainage areas of less than a few hundred square miles and for average recurrence intervals not longer than 100 years. Probably the error from using the same curve of 72-hr precipitation for drainage basins of all sizes is partly compensated by the error in using the duration of 72 hours for precipitation on all basins.

The frequency curve of 72-hr precipitation was combined with the probability table which showed the probability of a given depth of runoff being

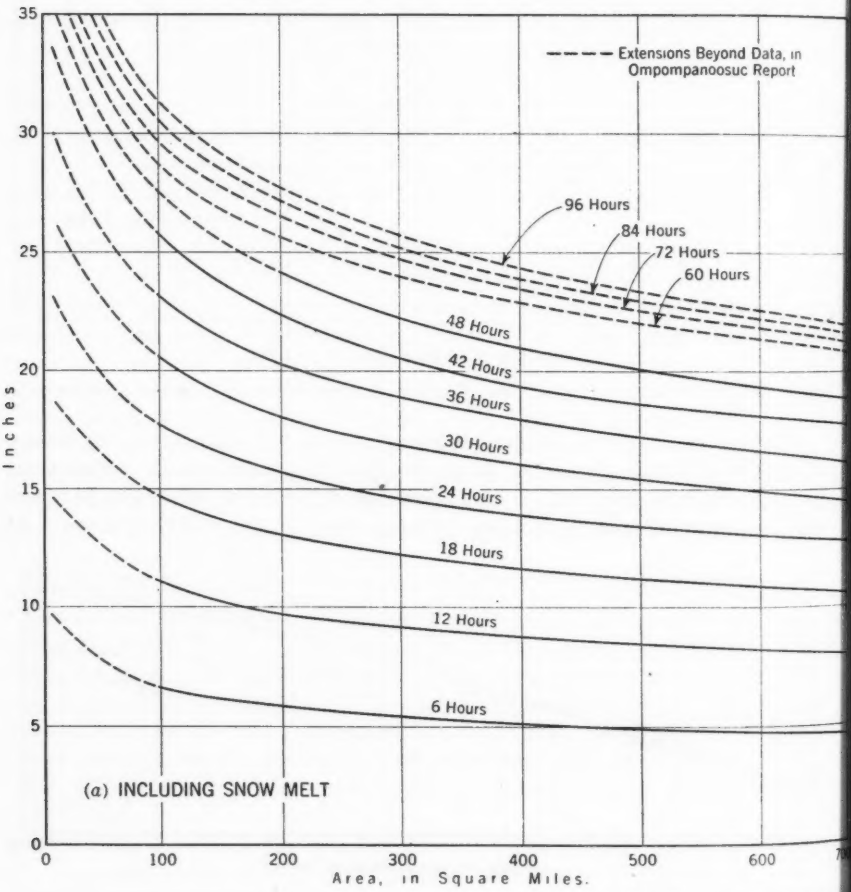
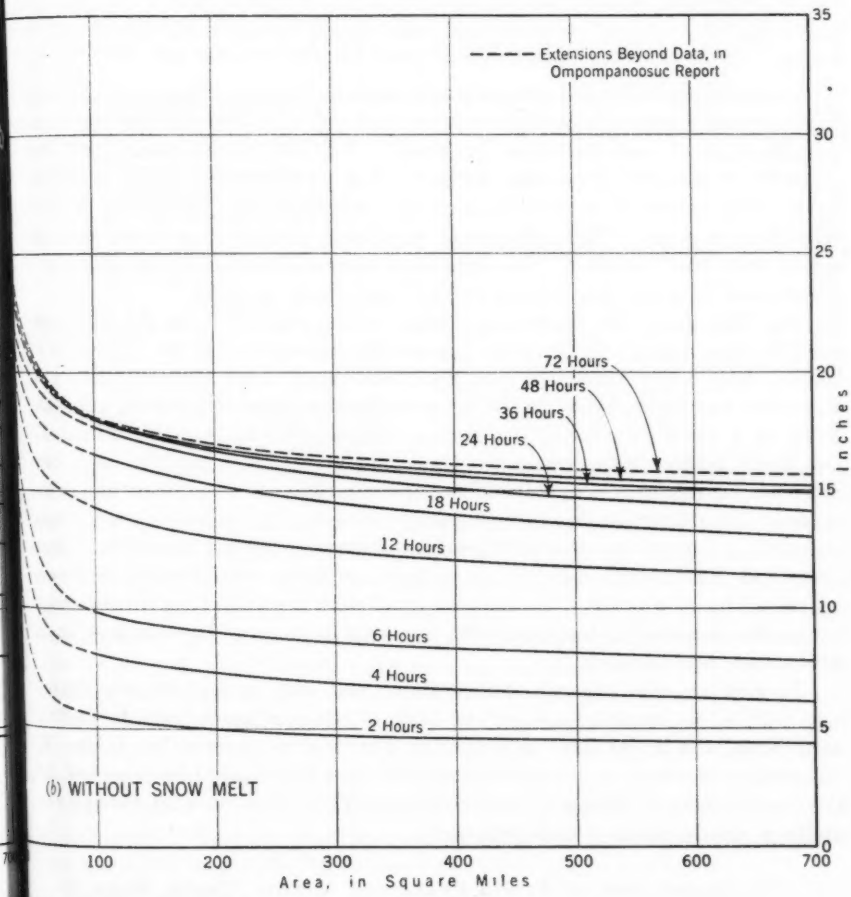


FIG. 8.—MAXIMUM RAINFALL BA

generated by any given depth of precipitation, and a frequency curve of depth of runoff obtained. This curve is shown as curve D in Fig. 7.

Frequency curves of depth of runoff were prepared for ten streams in Massachusetts. They have the same limitations as frequency curves of discharge, owing to shortness of record. Three of these drainage areas are widely separated, and for this study the records may be combined and used as one

continuous record of a length equal to the sum of the three individual records. These areas are Taunton River at State Farm, Mass., Priest Brook near Winchendon, Mass., and Middle Branch of Westfield River at Goss Heights, Mass., whose combined length of record is 69 years. Curve A in Fig. 7 is the frequency curve for this combined record. On the assumption that floods of the magnitude of those of March, 1936, and September, 1938, on the Middle Branch of Westfield River and Priest Brook might not have occurred during the period



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of record, these have been omitted in determining frequency curve B. The floods of 1936 and 1938 on Taunton River at State Farm were not exceptionally high.

Curves A and B differ widely for the larger recurrence intervals, and illustrate the possible influence of one or two extremely large floods in a comparatively short period of record. These curves represent the probable extremes

of a true frequency curve. Curve D shows lower depths of runoff at short recurrence intervals because some winter storms having less than 1.5 in. of precipitation were followed by as much as 2 in. of runoff.

The depths of runoff at recurrence intervals of 15, 100, and 1,000 years were estimated from curves A, B, and D. These depths were used later, with assumed time distributions of the runoff, to compute flood peaks. After a study of these peaks, the depths of runoff were revised slightly, and curve C, Fig. 7, represents these revised depths.

Maximum Flood Runoff

In the report of the Ompompanoosuc Basin in Vermont,[8] maximum possible rates of precipitation during both summer and winter storms and the maximum possible rates of snow melt were computed. Fig. 8(*b*) was prepared from that report.[16] Extensions from data shown in that report are indicated by dashed lines. One extension is from rainfall on a drainage area of 100 sq miles to rainfall at a point. The estimate of maximum rainfall at a point was also taken from that report.[17] The extension from a duration of 48 hours to a duration of 72 hours was determined by logarithmic plotting.

Fig. 8(*a*) shows the maximum possible winter rainfall[18] plus the maximum possible snow melt.[19] Extensions beyond the data available are indicated by dashed lines. The maximum possible quantity of water accumulated as ice and snow was estimated to be 10 in. on a drainage area of 1,000 sq miles and 16 in. on a drainage area of 10 sq miles. The detailed basis of the estimates is not given herein; but, during the time that stream-flow records have been collected, as much as 6 to 9 in. of precipitation have accumulated as ice and snow on some drainage basins in Massachusetts during winters when the total snowfall was much less than the greatest observed seasonal snowfalls. However, if the maximum possible amount of accumulated precipitation were overestimated by as much as 6 in., the computed peak discharges for the maximum winter floods would be less than 10% too large for most of the drainage basins which were investigated.

To any one who has not studied snow melt and amount of precipitation accumulated as ice and snow on the basis of the maximum possible limits of occurrence, the limits used in preparing Fig. 8(*a*) may seem too large. By "maximum possible" is meant not the maximum that should be expected, but the maximum that could not be exceeded under any possible, even though very unlikely, combination of circumstances.

Determination of Flood Peaks for Minor, Major, Rare, and Maximum Floods

The peak discharges of certain defined types of floods have been determined for several streams for which adequate discharge records are available. This

[16] "A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin above Union Village, Vt.," U. S. Weather Bureau, 1940, Figs. 36 and 44 (unpublished).

[17] *Ibid.*, Fig. 24.

[18] *Ibid.*, Table VII.

[19] *Ibid.*, Fig. 9.

was accomplished by determining for each basin the depth of runoff in inches for each type of flood, assuming a distribution of this runoff, and computing the flood hydrograph by means of the unit hydrograph.

Minor Flood Peaks.—For the purpose of this paper, a minor flood is defined as an approximation of a flood which would be equaled or exceeded on an average of once in 15 years. The depth of runoff for a minor flood has been found to be fairly uniform for drainage basins in Massachusetts. However, the depths of runoff do vary with physical characteristics. The best correlation of depth of runoff is with mean altitude above sea level. This relationship is shown in Fig. 9 where depth of runoff for minor floods is plotted against

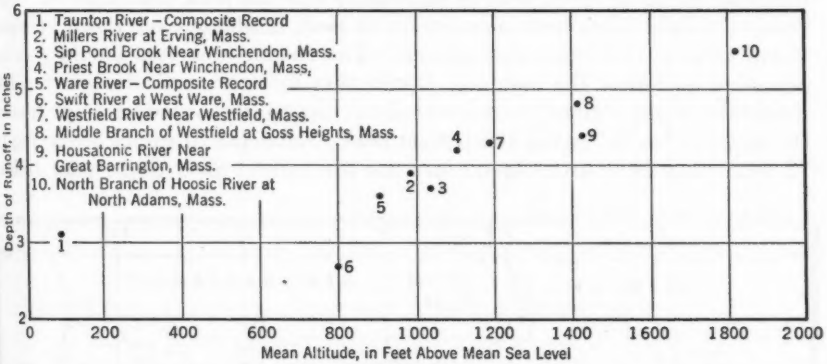

FIG. 9.—RUNOFF EQUALED OR EXCEEDED ON AN AVERAGE OF ONCE IN FIFTEEN YEARS

mean altitude above sea level. The points were obtained from frequency curves of depth of runoff for drainage areas having about twenty-five years of record, except No. 10, which has eight years of record. To make this relatively short record comparable to the longer records at other stations, it has been adjusted by the proportion which the frequency curve based on the last eight years of a longer record bears to a curve based on the complete record. Although the points scatter, and the relation below an altitude of 800 ft is not defined, the runoff definitely increases with an increase in altitude. The runoff for Swift River at West Ware, Mass., appears low in comparison with the other records, and was given very little weight in determining the correlation. The increase in runoff with an increase in the altitude of the basins may be attributed to factors other than the altitude, such as soil types, land slopes, and temperature, but these factors probably correlate fairly well with altitude.

The surface area of lakes, ponds, and reservoirs within a basin correlated to some extent with depth of runoff. No other correlation between depth of runoff and physical characteristics was found.

From the foregoing data, and Fig. 7, the final equation for depth of runoff for a minor flood was determined to be:

$$D = 3.6 - 0.3\,a + 0.001\,E \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which D is depth of runoff in inches; a is the water surface of lakes, ponds, and reservoirs expressed as a percentage of the drainage area; and E is the

difference, disregarding negative quantities, between the average altitude of the basin in feet above mean sea level and 800 ft.

The depth of runoff for a 72-hr storm was distributed by successive 6-hr increments of 3%, 5%, 5%, 10%, 10%, 20%, 34%, 10%, and 3%. For some areas, the 12-hr period covered by the two largest 6-hr increments was subdivided into 2-hr increments, 2%, 6%, 12%, 24%, 8%, and 2% of the total depth. The assumed runoff pattern has been determined from a study of precipitation and discharge records of Massachusetts and near-by areas, and a study of the distribution used by the Corps of Engineers for design floods.

Peak discharges computed from the depth-of-runoff formula and the distribution for 72-hr runoff were in agreement with the peak discharges of 15-yr floods obtained from frequency curves of peak discharges for those streams where these curves were well defined. The runoff, however, should not be presumed to follow the assumed distribution for any particular storm; nor should the depth of runoff be considered a representative value for 15-yr floods. On drainage basins having sharp flood peaks, the depth of runoff occurring in 72 hours may have little significance, for the highest flood peaks may follow

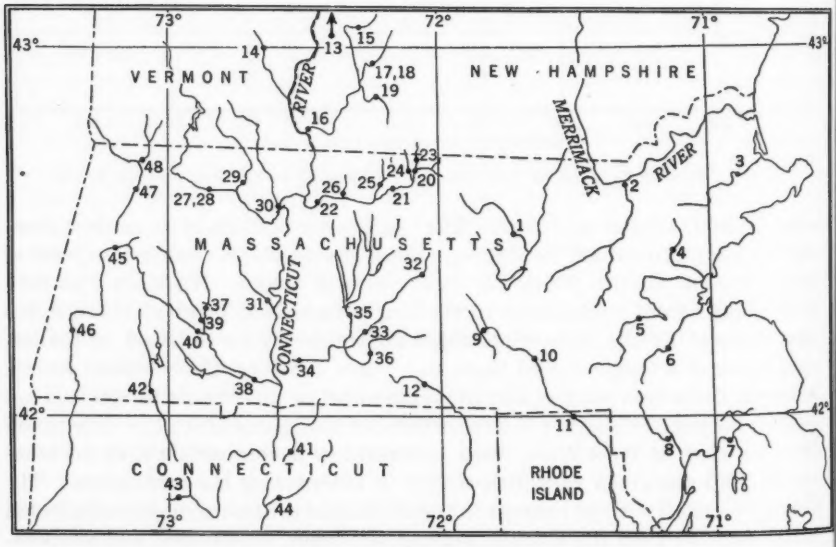

FIG. 10.—KEY TO GAGING STATIONS IN TABLE 1

short, intense periods of rainfall. Depths and distribution of runoff, as used in this study, are interdependent. The depths of runoff which have an average recurrence interval of 15 years may not be the same as the average depths of runoff associated with peaks of 15-yr floods. The distribution used here is only a means, uniform for all basins, of converting depth of runoff to discharge hydrographs having peaks which may be expected to occur with the same frequency as the depth of runoff.

The discharge hydrograph for a minor flood on each stream was computed by applying unit hydrographs to the 6-hr increments of depth of runoff assumed for the storm, and a base flow of 1 cu ft per sec per sq mile was included. The peaks are listed in Table 1 (and Fig. 10) together with the peaks of other classes of floods and the physical characteristics of the basins.

Major Flood Peaks.—A major flood is defined as an approximation of a flood that would be equaled or exceeded on an average of once in 100 years. The final equation for the depth of runoff was determined to be:

$$D = 6.1 - 0.35\,a + 0.001\,E \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The effect of lake and reservoir surface on the major floods as compared with minor floods was assumed to be slightly greater as measured in inches but to be considerably less as measured in percentage of the depth of runoff, and the effect of mean altitude was assumed to be the same as for minor floods.

A slightly more concentrated runoff distribution was assumed for the major flood than for the minor flood. Successive 6-hr increments of 2%, 3%, 5%, 10%, 10%, 20%, 38%, 10%, and 2% of the depth of runoff were used. For some areas, the 12-hr period covered by the two largest 6-hr increments was subdivided into successive 2-hr increments of 2%, 6%, 12%, 25%, 10%, and 3%. The discharge hydrographs for major floods on all streams were computed by the unit hydrograph method, and the peaks are listed in Table 1, Cols. 13 to 16. A base flow of 1 cu ft per sec per sq mile is included.

Rare Flood Peaks.—A rare flood is defined as an approximation of a flood that would be equaled or exceeded on an average of once in 1,000 years. The depth of runoff for rare floods was computed from the equation:

$$D = 8.9 - 0.4\,a + 0.001\,E \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The first term on the right-hand side of Eqs. 4 and 5 was based on curve C of Fig. 7. The effect of lakes and ponds on rare floods was assumed to be slightly greater than their effect on minor or major floods; and the effect of altitude on rare floods was assumed to be the same as its effect on other classes of floods.

Successive 6-hr increments of runoff of 2%, 3%, 5%, 8%, 10%, 20%, 40% 10%, and 2% of the total depth of runoff were used, and for some areas the two largest increments were subdivided into successive 2-hr increments of 2%, 6%, 12%, 27%, 10%, and 3%. For rare floods the discharge hydrographs were computed by the unit hydrograph method in the same manner as for minor and major floods. The computed peaks are listed in Table 1 and are plotted in cubic feet per second per square mile in Fig. 11. A base flow of 1 cu ft per sec per sq mile is included.

The depths of runoff used in computing rare floods and the computed peaks of the rare floods were compared with the greatest depths of runoff and the highest flood peaks that have been observed on streams in Massachusetts since stream-flow records have been collected. The computed figures seem consistent with the observed depths of runoff and the observed peak discharges. During the flood of September, 1938,[20] the depths of runoff and the peak dis-

[20] "Hurricane Floods of September 1938," *Water Supply Paper No. 867*, U. S. Geological Survey, 1940, pp. 14–35.

charges at gaging stations in the Millers River Basin except Moss Brook, and the Ware River Basin except Swift River at West Ware, were approximately the same as the depths and peaks which were computed for rare floods.

TABLE 1.—PHYSICAL CHA

No.[a]	Outlet of drainage area	Years[b]	PEAK DISCHARGE[c]		PHYSI
			Cu ft per sec	Date	M (sq mil
	(1)	(2)	(3)	(4)	(5)
1	North Nashua River near Leominster, Mass.	4	16,300	3-18-36	107
2	Concord River below River Meadow Brook, at Lowell, Mass.	3	3,790	7-29-38	405
3	Ipswich River near Ipswich, Mass.	10	2,610	3-15-36	124
4	Aberjona River at Winchester, Mass.	1	....	....	23.3
5	Charles River at Charles River Village, Mass.	2	3,170	3-19-36	183
6	Neponset River at Norwood, Mass.	1	....	....	35.2
7	Taunton River excluding Assawompsett Pond, at State Farm, Mass.	9	3,050	4-14-35	211
8	Wading River near Norton, Mass.	13	1,030	3-12-36	42.2
9	Blackstone River at Worcester, Mass.	15	2,520	3-18-36	31.3
10	Blackstone River at Northbridge, Mass.	1	....	....	139
11	Blackstone River at Woonsocket, R. I.	11	15,100	7-24-38	417
12	Quinebaug River at Westville, Mass.	1	....	....	93.8
13	Sugar River excluding Sunapee Lake, at West Claremont, N. H.	10	14,000	3-19-36	224
14	West River at Newfane, Vt.	14	52,300	9-21-38	308
15	Ashuelot River near Gilsum, N. H.	16	5,220	9-21-38	71.2
16	Ashuelot River at Hinsdale, N. H.	29	18,000	3-29-20	420
17	Otter Brook excluding Granite Lake, near Keene, N. H.	15	6,130	9-21-38	37.6
18	Otter Brook excluding Granite Lake and Ellis Reservoir, near Keene, N. H.	15	6,130	9-21-38	24.2
19	South Branch of Ashuelot River at Webb, near Marlboro, N. H.	18	5,960	9-21-38	36.4
20	Millers River near Winchendon, Mass.	22	8,500	9-22-38	83.6
21	Millers River at South Royalston, Mass.	1	....	....	186
22	Millers River at Erving, Mass.	24	29,000	9-22-38	370
23	Sip Pond Brook near Winchendon, Mass.	24	2,630	9-21-38	19.6
24	Priest Brook near Winchendon, Mass.	21	3,000	9-21-38	18.3
25	East Branch of Tully River near Athol, Mass.	22	5,140	9-21-38	49.6
26	Moss Brook at Wendell Depot, Mass.	23	1,540	3-19-36	12.2
27	Deerfield River at Charlemont, Mass.	26	56,300	9-21-38	362
28	Deerfield River excluding Harriman Reservoir, at Charlemont, Mass.	26	56,300	9-21-38	178
29	North River at Shattuckville, Mass.	1	....	....	88.4
30	Green River at Greenfield, Mass.	0	....	....	89.4
31	Mill River at Northampton, Mass.	1	....	....	53.2
32	Ware River at Cold Brook, Mass.	11	14,000	9-21-38	100
33	Ware River at Gibbs Crossing, Mass.	28	22,700	9-21-38	201
34	Chicopee River at Bircham Bend, Mass.	10	45,200	9-21-38	702
35	Swift River at West Ware, Mass.	28	7,590	3-19-36	188
36	Quaboag River at West Brimfield, Mass.	28	8,470	9-21-38	149
37	Westfield River at Knightville, Mass.	29	37,900	9-21-38	162
38	Westfield River near Westfield, Mass.	24	55,500	9-21-38	497
39	Middle Branch of Westfield River at Goss Heights, Mass.	30	19,900	9-21-38	52.6
40	West Branch of Westfield River at Huntington, Mass.	4	21,800	9-21-38	93.3
41	Scantic River at Broad Brook, Conn.	10	7,360	9-21-38	98.4
42	Farmington River excluding Otis Reservoir, near New Boston, Mass.	25	18,500	9-21-38	75.3
43	Burlington Brook near Burlington, Conn.	7	645	9-21-38	4.1
44	Hockanum River near East Hartford, Conn.	12	5,160	9-21-38	74.2
45	Housatonic River at Coltsville, Mass.	4	6,400	9-21-38	57.1
46	Housatonic River near Great Barrington, Mass.	25	11,500	9-22-38	280
47	Hoosic River excluding Hoosic Lake, at Adams, Mass.	8	5,080	9-21-38	31.8
48	North Branch of Hoosic River at North Adams, Mass.	9	9,980	11- ?-27	39.3

[a] See Fig. 10. [b] Years of record used in the study of flood peaks. [c] Maximum known peak discharge.

The maximum observed discharges and the computed peaks of rare floods are plotted in Fig. 11, which shows that on many streams the rare flood has not been remotely approached.

Maximum Flood Peaks.—A maximum flood is the computed flood that might be caused by the maximum possible precipitation, distributed according to the most destructive pattern, the losses from evaporation and infiltration being a

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PHYSI RACTERISTICS			COMPUTED FLOOD PEAKS IN CUBIC FEET PER SECOND								No.[a]
			From Flood Formulas				From Unit Hydrographs				
M (sq miles)	*s* (ft)	*a* (%)	Minor	Major	Rare	Maximum	Minor	Major	Rare	Maximum	
(5)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	
107	578	2.2	4,310	11,000	18,600	65,400	5,050	10,600	18,000	63,900	1
405	203	2.8	3,580	8,940	14,100	49,000	3,500	7,490	12,900	47,200	2
124	91	1.1	1,940	3,710	6,080	23,900	1,840	3,680	6,050	24,600	3
23.3	109	2.3	900	1,680	2,990	12,600	....	....	....	....	4
183	132	1.6	2,180	4,750	7,480	28,500	2,090	4,340	7,180	27,900	5
35.2	153	2.9	1,090	2,280	3,970	16,000	....	....	....	....	6
211	90	1.9	3,110	5,810	10,000	38,600	4,050	8,400	14,200	46,600	7
42.2	125	2.2	1,030	2,130	3,540	14,400	900	1,940	3,240	13,300	8
31.3	266	4.3	1,160	2,860	5,000	19,500	970	2,330	4,050	23,600	9
139	341	4.2	2,780	7,530	12,700	44,400	....	....	....	....	10
417	365	3.2	6,390	16,600	29,100	97,200	....	....	....	....	11
93.3	285	3.1	2,060	5,410	8,970	32,900	....	....	....	....	12
224	870	0.7	11,900	25,500	42,400	143,000	....	....	....	....	13
308	1,225	0.3	29,100	46,200	76,100	258,000	....	....	....	....	14
71.2	727	2.6	4,100	10,000	16,500	59,200	....	....	....	....	15
420	984	1.3	19,900	40,000	65,000	213,000	....	....	....	....	16
37.6	690	1.1	3,080	7,160	12,300	45,000	....	....	....	....	17
24.2	580	0.3	1,950	4,520	7,850	29,500	....	....	....	....	18
36.4	577	1.2	2,510	6,050	10,700	39,400	....	....	....	....	19
83.8	260	4.0	2,180	5,280	9,320	34,600	2,440	4,990	7,920	29,700	20
186	285	3.0	3,680	9,000	15,900	55,800	....	....	....	....	21
370	498	2.1	7,150	19,000	32,000	106,000	9,100	18,700	29,100	91,800	22
19.6	173	3.2	800	1,720	3,020	12,600	900	1,860	3,150	16,000	23
18.3	243	0.3	710	1,660	2,670	10,800	940	1,930	2,960	12,600	24
49.9	430	1.1	2,120	5,350	9,260	34,000	2,380	4,990	8,240	30,900	25
12.2	343	1.5	700	1,800	3,040	12,100	740	1,400	2,270	10,900	26
362	1,446	1.3	40,100	60,200	97,100	326,000	....	....	....	....	27
178	1,192	0.2	23,400	35,700	61,400	212,000	....	....	....	....	28
88.4	978	0.2	9,890	17,500	30,300	107,000	....	....	....	....	29
89.6	868	0.4	6,690	13,800	22,500	79,600	....	....	....	....	30
53.4	730	0.6	3,950	8,860	14,900	53,700	....	....	....	....	31
100	315	2.4	2,550	6,450	11,300	40,800	3,620	7,490	11,900	42,700	32
201	510	1.5	4,420	12,200	19,500	65,700	5,720	12,000	23,400	83,800	33
702	668	1.4	14,900	36,400	60,500	193,000	13,500	34,000	61,600	227,000	34
188	410	0.9	3,980	10,400	17,300	59,500	5,070	11,400	19,100	64,700	35
149	460	2.6	3,380	9,410	15,400	53,300	1,930	4,080	7,960	37,300	36
162	966	0.2	12,700	23,300	38,800	133,000	14,700	27,500	41,700	112,000	37
497	1,075	1.0	30,900	55,700	93,600	305,000	57,000	69,700	111,000	292,000	38
52.8	998	0.05	6,620	11,600	18,800	67,900	6,490	12,100	21,100	82,100	39
93.1	1,020	1.0	9,900	18,600	31,200	112,000	7,780	14,700	23,100	77,900	40
98.4	387	0.2	2,750	6,760	11,200	40,100	....	....	....	....	41
75.9	728	0.8	5,690	12,400	21,900	78,000	7,440	14,400	24,900	92,900	42
4.1	180	0.05	380	700	1,230	5,470	....	....	....	....	43
74.5	372	1.8	2,070	5,500	9,170	33,400	....	....	....	....	44
57.1	648	1.0	3,850	8,980	15,700	56,200	4,240	7,780	12,000	38,800	45
280	712	1.7	8,310	20,700	33,200	109,000	9,920	18,300	29,300	81,300	46
31.8	860	0.05	5,450	9,400	17,000	63,300	4,300	7,680	13,600	71,700	47
39.1	965	0.2	6,910	11,900	21,300	78,500	5,180	10,000	16,400	76,700	48

eak discharge

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minimum, and the discharge occurring in accordance with a unit hydrograph having an extremely high peak. The maximum flood (if it occurs during the winter) is a similarly computed flood which is based on the additional assump-

tions of the largest possible water content in accumulated snow and ice, the fastest rate of melting, and the worst combination of snow melt and precipitation.

Increments of maximum possible precipitation during summer storms were taken from Fig. 8(*b*) by 6-hr periods for each drainage basin. The maximum

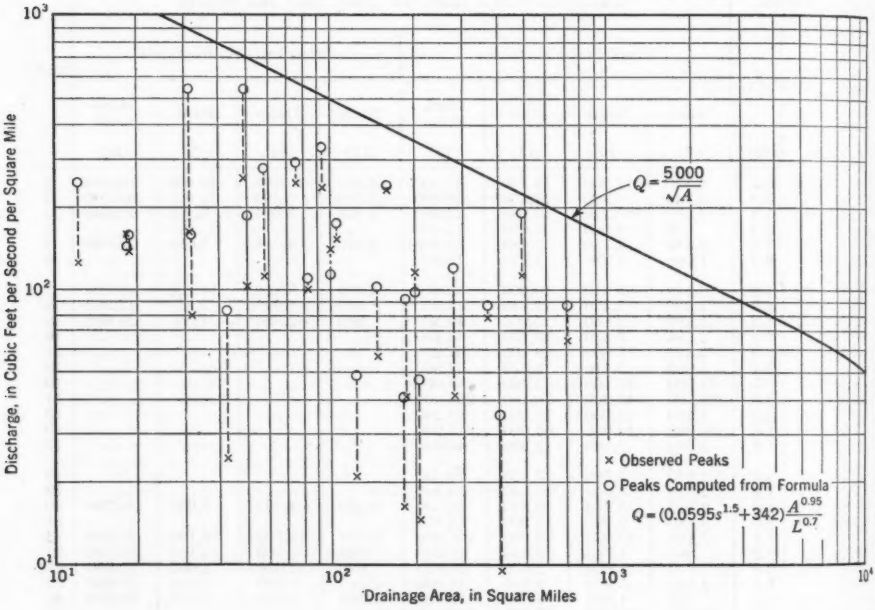
Discharge, in Cubic Feet per Second per Square Mile
Drainage Area, in Square Miles
$Q=\frac{5000}{\sqrt{A}}$
× Observed Peaks
○ Peaks Computed from Formula
$Q=(0.0595s^{1.5}+342)\frac{A^{0.95}}{L^{0.7}}$

FIG. 11.—RARE FLOOD PEAKS

6-hr increment was subdivided into 2-hr increments for the four streams whose flood hydrographs have the sharpest peaks. The 6-hr increments were arranged in the order which would produce approximately the highest flood peaks. The largest increment was preceded in order by the second, fourth, sixth, eighth, tenth, and twelfth largest increments. The largest increment was followed in order by the third, fifth, seventh, ninth, and eleventh largest increments.

When precipitation of high intensity falls on saturated soil, the losses by infiltration probably are small. Also, the precipitation which is called the maximum possible may not contain sufficient factors of safety, or the assumed distribution of the precipitation may not be that distribution which would cause the highest possible peak discharge. For these reasons a runoff factor of 100% was used.

Increments of winter precipitation plus snow melt were taken from Fig. 8(*a*) by 6-hr periods for each drainage basin. These increments were arranged to precede and to follow the maximum increment in an order similar to that used for the maximum possible summer storms. A runoff factor of 100% was used

for the drainage basins in the Providence, Thames, and Connecticut river valleys. Some appreciable loss by infiltration and evaporation must occur on the coastal basins, from which the discharge is slow. For these basins a runoff factor of 90% was assumed to be the maximum possible for winter floods.

Unit hydrographs were determined and the peaks of the maximum floods were computed in the same general way as those for the other classes of floods. Base flow of 2 cu ft per sec per sq mile was added to the computed net peaks of both the maximum winter and the maximum summer flood peaks. The lower one of the two computed peaks for each basin was discarded from further consideration, and the higher one was listed in Col. 16, Table 1. The maximum summer flood was higher or almost as high as the maximum winter flood for all drainage basins except the Concord River below River Meadow Brook, at Lowell, Mass., the Ipswich River near Ipswich, Mass., the Charles River at Charles River Village, Mass., the Taunton River excluding Assawompsett Pond, at State Farm, and the Millers River at Erving, Mass.

The maximum flood, as the term is used in this paper, is entirely different from a design flood or a maximum probable flood. It is that computed flood which is expected to equal or exceed the flood resulting from the worst combination of flood-producing conditions. It contains factors of safety for the inadequacy of past records and for ignorance both of relationships between cause and effect and the physical limits of natural hydraulic phenomena. Any design based on less than the maximum flood peak should be assumed to involve some risk. The remoteness of the probability that a flood peak even close to the size of the peak of the maximum flood will occur during the existence of any structure is indicated by the great difference between the computed peaks of the rare floods and those of the maximum floods.

The reason for the large size of the maximum flood peak can be shown best by contrasting that peak with a flood having the order of magnitude of a maximum probable flood. This flood might follow precipitation that was 80% of the maximum, the distribution might have an effect on the peak which was 85% of the maximum, the runoff factor might be 85%, and the unit hydrograph might have a shape which would have 75% of the maximum effect on the computed flood peak. This flood would then have a peak discharge which would be about 45% of the peak discharge of the maximum flood. A similar rough analysis applied to winter floods would show the peak of the maximum probable flood to be an even smaller percentage of the peak of the maximum winter flood.

Physical Characteristics

The physical characteristics of more than one hundred drainage basins in New England were computed by the personnel of the Work Projects Administration under the supervision of the Boston district of the U. S. Geological Survey. Of these basins, forty eight were in Massachusetts, although not all of them were entirely separate basins because some overlapped. Except for the type of vegetal cover, which was computed from information supplied by the National Resources Committee, the following physical characteristics were computed from topographic maps published by the U. S. Geologic Survey:

Areas.—(1) Drainage area; (2) area of lake, reservoir, and pond surfaces; and (3) area of swamps.

Altitude.—(1) Maximum altitude; (2) minimum altitude (altitude at the outlet); (3) median altitude (altitude at which 50% of the drainage area is higher and 50% is lower); and (4) mean altitude.

Slopes.—(1) East-west land slope; (2) north-south land slope; (3) average land slope; (4) slope of principal streams (a principal stream being defined as a watercourse that drains more than 10% of the total area of the basin); and, (5) slope of tributary streams (tributary streams being defined as all watercourses contributing to a principal stream).

Configuration of Basin and Streams.—(1) Length of principal streams; (2) stream density (total length of all streams divided by the drainage area); (3) average distance uniformly distributed runoff must travel to the outlet; (4) length of basin (twice average distance water must travel); (5) width of basin (drainage area divided by length of basin); (6) form factor (length of basin divided by width of basin); and (7) branch index (length of stream channels, selected according to an arbitrary criterion, divided by length of basin).

General Type of Vegetal Cover.—(1) Percentage of cropped land; (2) percentage of wooded land; and (3) percentage of grass land not cropped.

Correlations with Flood Peaks

The peak discharge of a flood probably would be affected by any large change in any one of these basin characteristics even though all the other characteristics remained constant; but the change might be very slight. Also a change in one characteristic is usually associated with changes in other characteristics. Therefore, as the computed flood peaks are not exact and the relationships among the characteristics are complex, the effect of any particular characteristic usually cannot be determined accurately. Thus, the results of correlations may not really represent laws of cause and effect, so the formulas may not be applicable to regions other than Massachusetts or to ranges in any of the characteristics which are much beyond the limits included in this study. The correlations show, however, that changes in certain characteristics were associated in a definite way with changes in the size of the flood peaks.

Each computed physical characteristic was considered as a possible basis for correlation equations except the maximum altitude, the minimum altitude, and the east-west and north-south land slopes. Correlations between peak discharge and the physical characteristics for each type of flood were determined first by plotting on logarithmic paper the peaks of that type of flood against the drainage area. A straight line was used to represent the relation between peak discharge and drainage area. The ratio r_1 between each peak discharge and the peak discharge which would plot on the straight line was computed. Correlations of ratios r_1 with stream slopes, land slopes, and mean elevations above the outlet were investigated. Before any one of these characteristics was plotted against the ratios r_1, any correlation between the characteristic and drainage area was eliminated. To do this, the characteristic was plotted against the drainage area, and a curve showing the relationship was drawn. Ratios r_2 of the departure of the characteristic from this curve were computed. Then ratios r_1 were plotted against ratios r_2 and a curve was drawn to fit the plotted points. Ratios r_3 of the departure of the plotted

points from this curve were computed. Next these ratios r_3 were correlated with another characteristic from which any correlation with a characteristic previously included in the correlation had been eliminated. When the remaining ratios no longer showed a correlation with a basin characteristic, the correlations already found were adjusted slightly to make them fit the base data as closely as possible.

The formulas representing the best correlations which were found are as follows: For minor floods—

$$Q = (0.000036\, s^{2.4} + 124)\, \frac{M^{0.95}}{a^{0.04}\, L^{0.7}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

and for major floods—

$$Q = (0.0344\, s^{1.5} + 200)\, \frac{M^{0.85}}{L^{0.5}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

for rare floods—

$$Q = (0.0595\, s^{1.5} + 342)\, \frac{M^{0.95}}{L^{0.7}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6c)$$

and for maximum floods—

$$Q = (0.128\, s^{1.6} + 1{,}800)\, \frac{M^{0.9}}{L^{0.7}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6d)$$

In these equations Q is the peak discharge in cubic feet per second; a is the percentage that lake, pond, and reservoir surface is to the total drainage area; s is the median altitude of the drainage basin in feet above the outlet; M is the drainage area in square miles; and L is the average distance, in miles, which water from runoff uniformly distributed over the basin must travel to the outlet. Thus M is a measure of the volume of water to be discharged; s is a measure of the fall of the water; and L is a measure of the distance that the water travels to the point of discharge. In addition, a in Eq. 6a is a rough measure of the effect of storage in lakes and reservoirs on the quantity of water discharged or on the shape of the hydrograph. The form of the equation indicates that a must not be zero. It should always be assumed to equal or exceed 0.05% even though the topographic map shows little or no water surface of lakes, ponds, or reservoirs. As a is not an entirely satisfactory measure of the storage, flood flows should be routed through the large lakes or reservoirs or those which have a large portion of the flow passing through them. The absence of a in the formulas for large floods (Eqs. 6b, 6c, and 6d) should not be interpreted to mean that lakes and reservoirs have no effect on the peak discharge of these floods. These floods should also be routed through the larger lakes or reservoirs. Perhaps no correlation was found because a correlates to some extent with either L or s, or the correlation is obscured by the effect of other basin characteristics or by inaccuracies in computed flood peaks.

The measures of basin characteristics, although the most practicable, are not entirely satisfactory. Average stream and land slopes, altitudes, and lake and swamp areas were used without determining the distribution of them in the basins. Steep slopes on a few tributaries may increase the average slope considerably, yet these slopes may have little effect on peak discharges. Steep slopes or abnormally high altitudes in that part of a basin upstream from a lake may affect the average land slope and mean elevation of the basin materially; but the lake may decrease the peak discharge so much that the peak outflows

from the lake would be little different if the slopes and elevations upstream from the lake were greatly reduced. A lake on the headwaters of a flashy stream may have no noticeable effect on flood peaks, but a similar lake on the main channel near the outlet may greatly decrease the peak discharges. Consideration should be given to these facts in the study of any particular basin.

General type of vegetal cover did not correlate with peak discharge. Probably no correlation should be expected in Massachusetts where little of the land is either bare or in cultivated crops and where nearly all of the basins have a large proportion of woodland. Area of swamps seemed to correlate somewhat with stream slopes, with s, and with peak discharges. However, topographic maps based on surveys made at different times usually show different swamp areas. Median altitude usually is about the same as the mean altitude, and the differences between the two did not seem to be associated with a consistent variation in peak discharges. Average land slope is a fairly satisfactory measure of topography for use in correlation equations, but it did not correlate with peak discharges as well as mean altitude above the outlet. Slopes of principal streams and tributary streams are important basin characteristics, but the division between principal and tributary streams is arbitrary, and the average slope of either is not a very satisfactory measure of channel storage or of time of travel to the outlet. Stream density does not change much from place to place in Massachusetts, and the changes which do occur tend to correlate with land slopes. Neither the branch index nor the length of principal streams seemed to be as good a measure of the concentration characteristics of drainage basins as either the form factor or the average distance that water has to travel to the outlet. Length of basin, width of basin, form factor, and average distance water has to travel to the outlet are all interdependent, and as good correlations can be obtained with one as with any of the others. However, the average distance water must travel to the outlet has a simple and direct meaning; so it was used.

Mean altitude above sea level does not appear in the formulas, although it was included in the equations for depth of runoff. However, it correlates fairly well with s, the mean altitude above the outlet of the basin, so that by including s in the formulas, the effect of mean altitude above sea level is not neglected.

Minor, major, rare, and maximum flood peaks were computed with slide-rule accuracy for 48 basins from the formulas and are listed in Table 1 (Cols. 9 to 12). Table 1 also shows the physical characteristics that were used in the final formulas, the flood peaks computed from unit hydrographs, and the maximum observed or known flood peaks published by the U. S. Geological Survey.[21] No maximum known peak discharge is given if the period of record is short and no large flood discharge from the basin has been determined.

The points for which peak discharges were computed, usually sites of gaging stations operated by the U. S. Geological Survey, are shown in Fig. 10. Numbers on the map correspond to the numbers given in Table 1.

Frequency curves of peak discharge for three streams are given in Fig. 12. Curves such as these form the best available check on the computed peak dis-

[21] *Water-Supply Paper No. 871*, U. S. Geological Survey, 1939.

charge determined from the formulas. However, in determining the curve for Fig. 12(*b*), due consideration must be given to the extreme size of the flood of September, 1938. The frequency curves shown in Fig. 12 are not identical with those shown in Fig. 1 for the same stations, since the values obtained from the formulas are used in making the extensions beyond the observed data.

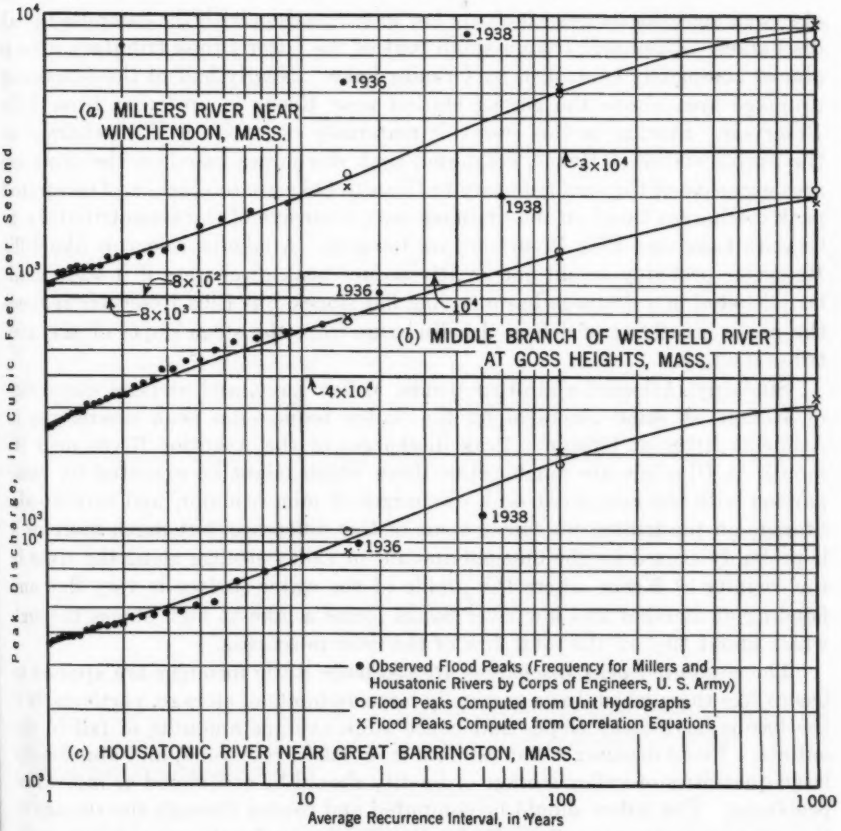
Peak Discharge, in Cubic Feet per Second
(a) MILLERS RIVER NEAR WINCHENDON, MASS.
1938
1936
3×10⁴
8×10²
8×10³
(b) MIDDLE BRANCH OF WESTFIELD RIVER AT GOSS HEIGHTS, MASS.
4×10⁴
(c) HOUSATONIC RIVER NEAR GREAT BARRINGTON, MASS.
• Observed Flood Peaks (Frequency for Millers and Housatonic Rivers by Corps of Engineers, U. S. Army)
○ Flood Peaks Computed from Unit Hydrographs
× Flood Peaks Computed from Correlation Equations
Average Recurrence Interval, in Years

FIG. 12.—FREQUENCY OF FLOOD PEAKS

Flood peaks computed from the formulas are too high for Quaboag River at West Brimfield, Mass., because of unusual valley storage in the basin. The slope of the river surface from West Brimfield upstream to Warren is steep, but for about 10 miles above Warren the stream is very flat. Large areas in this part of the valley are flooded at high stages, and the lake thus formed reduces the peak discharges at West Brimfield materially. Short periods of intense precipitation sometimes cause two distinct peak discharges at West Brimfield. The first one is a sharp peak caused principally by the runoff from the area downstream from Warren which is about 10% of the total. The second is a smooth, flat peak caused principally by runoff from the area above Warren.

The formulas should be used with care on streams like the Ashuelot River in New Hampshire and its tributaries. The South Branch of Ashuelot River near Marlboro, N. H., has no large amount of natural or artificial storage in the basin, and the computed peaks are consistent with the observed peaks. Otter Brook, another tributary of the Ashuelot River, near Keene, N. H., the Ashuelot River at Gilsum, N. H., and the Ashuelot River at Hinsdale, N. H., have observed peak discharges which are low in comparison with the computed peak discharges. Discharge from a small part of the Otter Brook tributary area is almost completely controlled by Granite Lake. Nearly half of the remaining drainage area above the gaging station near Keene is upstream from Ellis Reservoir. Storage in this reservoir materially decreases peak discharges at the gaging station. Hence, computed peak discharges based on the drainage area exclusive of the area tributary to Granite Lake are too high, and computed peak discharges based on the drainage area, exclusive of the areas tributary to Granite Lake and Ellis Reservoir, are too low. A lake or reservoir like Ellis Reservoir probably would not greatly reduce peak discharges if it were similarly located in a drainage basin having flat slopes, but Ellis Reservoir reduces the peaks greatly, and delays the discharge from the steep slopes of the area tributary to it.

Similarly, Ashuelot and other ponds, Millen Lake, and unusual quantities of storage at some places along the valley reduce the peak discharges of Ashuelot River at Gilsum. Peak discharges of the Ashuelot River near its mouth at Hinsdale are much below those which might be expected by comparison with the computed peak discharges of minor, major, and rare floods. Storage on the tributaries causes some of this difference, but much more of it is probably caused by the unusual amount of valley storage along the river in the vicinity of Keene where the profile of the water surface is very flat and flooding of marshes and low river banks forms a lake at high stages through which about 80% of the total flow of the river must pass.

The computed peak discharges are too large if the formulas are applied to basins like these which have unusually large amounts of storage, particularly if the basins have steep slopes and hence large average amounts of fall to the outlets. Flood discharges downstream from lakes, reservoirs, and abnormally large quantities of valley storage ordinarily should be considered as individual problems. The inflow should be computed and routed through the storage to the point downstream where discharges are required, allowance being made for inflow to the reaches through which the outflow is routed.

Peak Discharges for Ungaged Areas

To use the formulas in computing the different classes of flood peaks for a particular point on a stream, the basin characteristics, M, a, L, and s, must be computed with whatever degree of accuracy is required. The drainage area M can be readily determined by planimeter. Water-surface areas necessary for computing a can be obtained by measuring with a planimeter large surfaces and estimating small surfaces by comparison with a set of squares or rectangles of known area. Length L can be estimated directly from a map for small areas. Large areas, however, must be divided into small basins, for which the

distance of water travel to the point of discharge from the main area can be measured, and L computed by weighting according to the area of each subdivision. The median altitude can be computed by preparing a grid, on transparent paper, of squares representing areas of one quarter of a square mile or other convenient unit, laying this grid over the topographic map on which the basin is outlined, and estimating the average altitude of the area within each unit or fractional unit of area. The weighted average of these units will be the median altitude of the total area. The altitude of the outlet of the basin is subtracted from the median altitude to determine s.

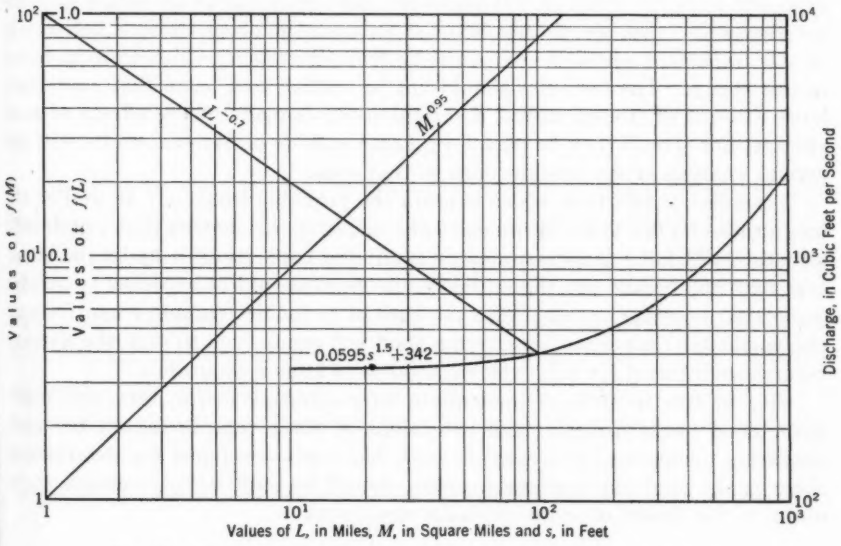

FIG. 13.—DIAGRAM FOR SOLVING EQ. 6c, THE FORMULA FOR RARE FLOODS

Fig. 13 is a diagram that can be used to determine peak discharges for rare floods. Similar diagrams can be prepared for other classes of floods, for convenience in computation.

CONCLUSIONS

The peak discharges of unit hydrographs increase as the peaks of the floods from which they were derived increase, and unit hydrographs developed from small floods should not be used without adjustment to compute the hydrographs of much larger floods.

Depths of runoff and the typical distribution of runoff can be combined with unit hydrographs to construct flood peaks that seem reasonably consistent with the frequency and magnitude of observed peaks. Depth of flood runoff does not vary appreciably with the size of drainage basin except for maximum floods, but it does increase with an increase in altitude, at least above 800 ft, and it decreases with an increase in the surface area of lakes and ponds.

Frequency curves of instantaneous peak discharges or of depths of runoff, even though they are based on 20 to 30 years of record, can be misleading,

especially if the records contain one or two very large floods. The frequency curve of the Connecticut River based on 100 years of record was changed radically by the inclusion of the flood of 1936.[22] Studies of frequencies made from records prior to some of the disastrous floods of recent years would have indicated that such floods should never have occurred.

Flood peaks can be correlated sufficiently well with physical characteristics of drainage basins to determine the peaks for ungaged areas more accurately than they can be computed from general flood formulas which involve only the drainage area, or the two factors of drainage area and shape of basin. However, the formulas given in this paper are not applicable to all ranges of characteristics and they should not be used for drainage areas which differ widely in any important physical characteristic from the basins which were included in this study. The formulas should not be applied to a basin that contains a large amount of storage unless a careful study is made of the effects of such storage; nor should they be used for an area outside of Massachusetts without careful analysis of stream-flow data in that area.

Formulas of this type which contain the probable frequency as well as the magnitude of a flood provide a means of evaluating the chances that a hydraulic structure will fail. Any assumed or computed capacity of a hydraulic structure that will handle less than a maximum flood should be expected to involve some risk, however remote. No measure of probable frequency can forecast the particular time when an extreme flood will occur even though the average recurrence interval for this extreme flood were known accurately.

Stream-flow records are inadequate for computing major, rare, and maximum flood peaks directly, and extensions of these records cannot be made precisely. However, a reasonable basis has been developed for determining these peaks, and the computed peaks should be sufficiently accurate to be useful in the design of some hydraulic structures.

Acknowledgments

The drainage basin characteristics, upon which much of the work in this study was based, were determined by the Work Projects Administration in the Boston office of the Geological Survey, by methods developed by the Division of Water Utilization of the Geological Survey, under the direction of R. W. Davenport, M. Am. Soc. C. E., chief.

The assistance of the Corps of Engineers, U. S. Army, in this study is gratefully acknowledged. This agency furnished many studies used in the investigation, including more than one hundred unit hydrographs, several frequency curves, and rainfall distribution and frequency data.

The material in this paper is drawn to a large extent from an unpublished report by the co-author, B. R. Colby, entitled "Flood Peaks on Massachusetts Streams." The study upon which this paper was based was conducted by Mr. Colby and others in the Boston office of the U. S. Geological Survey, under the direction of the co-author, H. B. Kinnison, district engineer. The writers are indebted to C. C. McDonald, Assoc. M. Am. Soc. C. E., for assistance in the preparation of this paper.

[22] *Proceedings*, Boston Soc. of Civ. Engrs., January, 1942, Fig. 33, p. 106.

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PAPERS

SHEAR AND BOND STRESSES IN REINFORCED CONCRETE

BY STANLEY U. BENSCOTER[1] AND SAMUEL T. LOGAN,[2]
JUNIORS, AM. SOC. C. E.

INTRODUCTION

Formulas in common use for computing shear and bond stresses in reinforced concrete beams are based on the assumption that the member is subjected to symmetrical bending without thrust. The object of this paper is to demonstrate the error involved in applying them to members that carry thrust, and to develop general formulas which are applicable to prismatic members with or without thrust. The necessity for such formulas becomes apparent upon realization that practically all members of rigid frames are subjected to thrust as well as bending. The paper provides a new viewpoint for the problem of shear and bond stresses in reinforced concrete and is offered for the purpose of promoting discussion on the problem.

NOTATION

The following letter symbols, adopted for use in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = area of cross section of steel;
b = width of a member;
c = distance from the centroid of a section to the fiber with maximum unit stress;
d = effective depth of a beam:
$k\,d$ = part of the depth that defines the distance from the compressive fiber to the neutral axis;

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by August 1, 1944.

[1] Associate Engr., Bureau of Aeronautics, Navy Building, Washington, D. C.
[2] 1st Lt., Corps of Engrs., U. S. Army, Camp Joseph T. Robinson, Ark.

[3] ASA—Z10a—1932.

αd = part of the depth that defines the distance from the compressive fiber to the compressive steel;
I = moment of inertia;
j = internal moment arm = $1 - \frac{k}{3}$;
k = a proportion of the effective depth (see d);
n = the modular ratio;
$p = \frac{A}{b\,d}$ = ratio of steel (compressive or tensile) to the sectional area of the beam;
Q = an integral defined by Eq. 4;
S = section modulus;
T = force acting on tensile reinforcement, dT being a small increment of such force;
u = unit bond stress;
V = total shear;
v = unit shearing stress;
x = a lateral distance, dx (see Eq. 1 and Fig. 1) denoting an element of width b;
y = a vertical distance measured from the centroidal axis: y_0 = the vertical distance between the centroidal axis and a point lying in the tensile region (see Figs. 1 and 2);
α = a proportion of the effective depth (see d);
Σ_o = perimeter of tensile steel.

Subscripts c and t used with the foregoing nomenclature denote "compression" and "tension," respectively. Subscript m denotes "maximum."

Beam Without Thrust

A differential slice of a beam, subjected to bending and shear, is shown in Fig. 1 with the forces that act upon the slice. The centroidal axis and the neutral axis coincide. The distances c_c and c_t are the distances from the centroidal axis to the extreme fibers. The forces acting on the tensile reinforcement are T and $T + dT$. Equilibrium of the differential length of the tension bar (or bars) requires that the bond stress u be given by:

$$u = \frac{1}{\Sigma_o}\frac{dT}{dx} \qquad (1)$$

On any horizontal plane between the neutral axis and the tension steel, such as is indicated by the distance y_0, the shearing stress v is given by:

$$v = \frac{1}{b}\frac{dT}{dx} \qquad (2)$$

From structural mechanics the well-known formula for shearing stress in a beam may be taken as

$$v = \frac{V\,Q}{I\,b} \qquad (3)$$

in which Q is defined by the integral,

$$Q = \int_{y_0}^{c_t} y\, dA \quad \ldots\ldots (4)$$

in which y is measured from the centroidal axis. For the case illustrated in Fig. 1, the quantity Q is merely the moment (about the centroidal axis) of the transformed tensile steel area and, hence, is given by

$$Q = n\, A_t\, c_t \quad \ldots\ldots (5)$$

in which A_t is the tensile steel area. Substitution of Eq. 5 into Eq. 3 gives

$$v = \frac{V\, n\, A_t\, c_t}{I\, b} \quad \ldots\ldots (6)$$

Two section moduli S_c and S_t may be defined for the beam as

$$S_c = \frac{I}{c_c}\,; \text{ and } S_t = \frac{I}{c_t} \quad \ldots\ldots (7)$$

The first modulus S_c may be divided into the bending moment to obtain the maximum compressive stress, and the second modulus may be used similarly

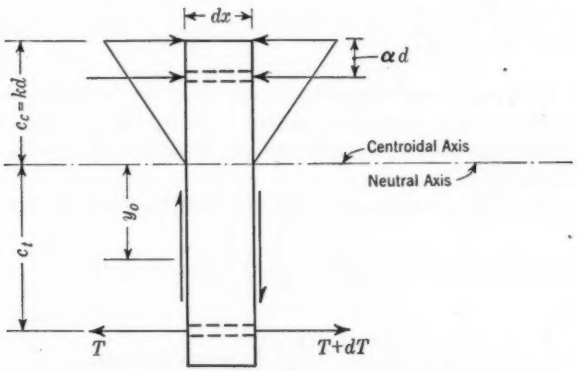

FIG. 1.—MEMBER WITHOUT THRUST

to compute the stress in the tensile steel. Introducing S_t into Eq. 6 gives the following formula for the shearing stress,

$$v = \frac{V\, n\, A_t}{b\, S_t} \quad \ldots\ldots (8a)$$

From Eqs. 1, 2, and 8*a* it may be seen that the corresponding formula for bond stress is

$$u = \frac{V\, n\, A_t}{\Sigma_o\, S_t} \quad \ldots\ldots (8b)$$

since

$$u = \left(\frac{b}{\Sigma_o}\right) v \quad \ldots\ldots (9)$$

The usefulness of Eqs. 8 depends upon having a knowledge of the section modulus S_t. The following formulas may be derived for S_c and S_t,

$$S_c = \frac{b\,d^2}{6}\left[\,2\,k^2 + 6\,p_c\,(n-1)\,\frac{(k-\alpha)^2}{k} + 6\,n\,p_t\,\frac{(1-k)^2}{k}\,\right] \ldots\ldots (10a)$$

and

$$S_t = \frac{b\,d^2}{6}\left[\,\frac{2\,k^3}{1-k} + 6\,p_c\,(n-1)\,\frac{(k-\alpha)^2}{1-k} + 6\,n\,p_t\,(1-k)\,\right] \ldots (10b)$$

Since the neutral axis is also the centroidal axis, an equation of moments of areas about this axis can be written as follows:

$$(k-\alpha)(n-1)\,A_c\,d + \frac{b}{2}\,(k\,d)^2 = n\,A_t\,d\,(1-k) \ldots\ldots\ldots (11)$$

Since Eq. 11 gives a linear relationship between A_c and A_t, it is a simple matter to eliminate either p_c or p_t from Eqs. 10. If p_t is eliminated the formulas for S_c and S_t become:

$$S_c = \frac{b\,d^2}{6}\left[\,k\,(3-k) + \frac{6\,p_c\,(n-1)(k-\alpha)(1-\alpha)}{k}\,\right] \ldots\ldots (12a)$$

and

$$S_t = \frac{b\,d^2}{6}\left[\,\frac{k^2\,(3-k)}{1-k} + \frac{6\,p_c\,(n-1)(k-\alpha)(1-\alpha)}{1-k}\,\right] \ldots\ldots (12b)$$

It is possible that reinforced concrete beams may be designed more conveniently by using the section modulus concept than by the methods now in common usage. This may be particularly true for designers who work alternatively with steel, timber, and concrete.

If a beam does not contain compressive steel, the formula for S_t becomes:

$$S_t = \frac{k^2\,(3-k)}{1-k}\left(\frac{b\,d^2}{6}\right) (\text{when } p_c = 0) \ldots\ldots\ldots\ldots (13)$$

From Eq. 11,

$$A_t = \frac{k^2}{n\,(1-k)}\left(\frac{b\,d}{2}\right) (\text{when } p_c = 0) \ldots\ldots\ldots\ldots (14)$$

Substituting Eqs. 13 and 14 in Eqs. 8:

$$v = \frac{3\,V}{(3-k)\,b\,d} = \frac{V}{b\,j\,d} \ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

and

$$u = \frac{3\,V}{(3-k)\,\Sigma_o\,d} = \frac{V}{\Sigma_o\,j\,d} \ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

in which $j = 1 - \frac{k}{3}$. These formulas, which are in common use, are seen to be in exact agreement with Eqs. 8 for beams without compressive reinforcement. When a beam contains compressive steel, Eqs. 8 give the correct formulas for v and u, whereas Eqs. 15 are somewhat in error.

Member with Thrust

In the case of a member carrying thrust and bending, the neutral axis and the centroidal axis do not coincide. The centroidal axis always lies within the compressive region, as indicated in Fig. 2. The formulas for v and u,

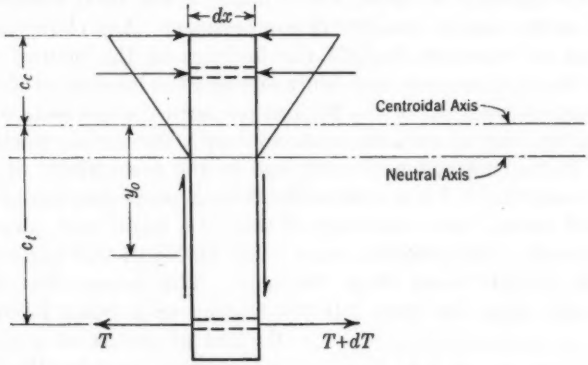

Fig. 2.—Member with Thrust

given by Eqs. 8, are also applicable to this case. The shear stress v will be the shear stress occurring at any point between the neutral axis and the tensile steel. This is the shear stress which is of interest in reinforced concrete design since it is equal in magnitude to the diagonal tensile stress.

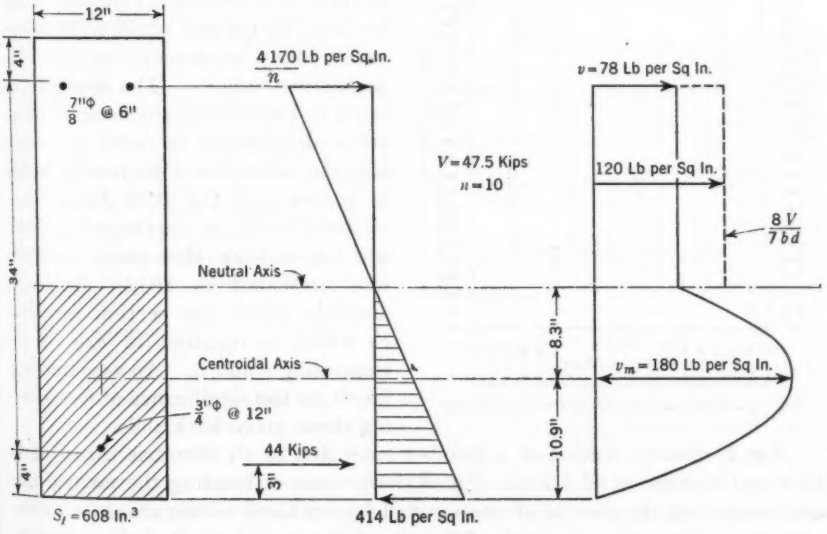

Fig. 3.—Internal Stress Distributions

In order that the use of Eqs. 8 may be clear it may be well to emphasize certain characteristics of the section properties of a member with thrust. The distance y in Eq. 4 and the distance c_t in Eq. 5 are measured from the centroidal

axis (not the neutral axis). The integral Q, for determining the shearing stresses in any elastic beam, is always computed about the centroidal axis. For given values of thrust and moment the neutral axis can be located. This determines the compressive region. The centroidal axis lies at the center of gravity of the transformed area which includes the total transformed steel area as well as the area of concrete in compression. Any change in the value of the thrust or moment changes the location of the neutral axis. This changes the transformed area and hence changes the location of the centroidal axis. More specifically, all of the flexural properties of the section (the transformed area, the neutral axis, the centroidal axis, the section moduli, and the moment of inertia) change with a change in the eccentricity of the thrust. When the eccentricity is known, all of the flexural properties can be computed. The case that occurs most commonly is that of a beam with constant thrust from end to end. The moments vary along the span and hence the eccentricity of the thrust varies along the span. This means that the flexural properties vary along the span. If the loading on a beam is changed, the flexural properties at a given section will change, assuming that the change in loading produces a change in the moment or thrust at the given section.

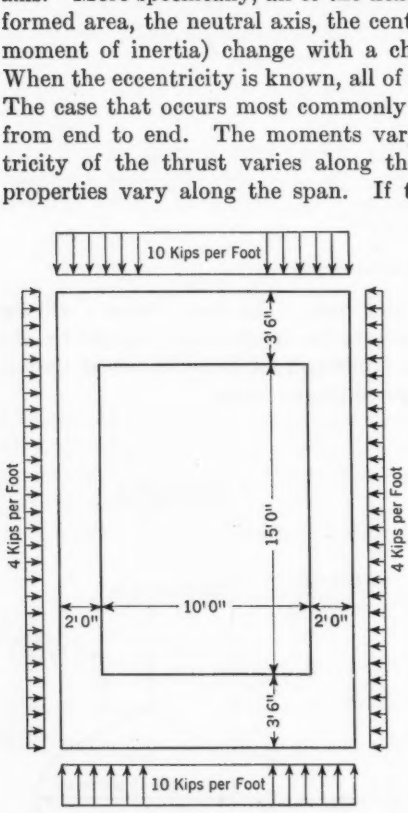

FIG. 4.—RECTANGULAR CONDUIT WITH LOADS

Due to the complicated algebraic relationships which arise in a stress analysis of a reinforced concrete member with thrust and bending, it is generally found convenient to use trial-and-error methods. The neutral axis must be located and the flexural properties determined in order to establish the allowable compressive stress in accord with the 1940 Joint Committee Code.[4] A convenient procedure for making this stress analysis was presented in 1942.[5] Such an analysis yields the section modulus S_t which is required in Eqs. 8 for computing v and u. An example will illustrate the significance of the shearing stress given by Eq. 8*a*.

Fig. 3 shows a section of a member subjected to an eccentric thrust of 44 kips and a shear of 47.5 kips. Since the process of locating the neutral axis and computing the normal stresses is well known these values are shown without discussion. The moment of inertia of the transformed section may be computed readily by straightforward procedures, and the section modulus S_t

[4] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 17, p. 73.

[5] "Thrust and Symmetrical Bending in Reinforced Concrete," by Stanley U. Benscoter, *Civil Engineering*, April, 1942, p. 214.

may be computed to be as shown. The shearing stress diagram is the feature of interest in this example. From the general formula (Eq. 3) it is possible to compute the shear stress at various points within the compressive region and thus to plot the shear-stress diagram. This diagram is of parabolic shape with a slight irregularity at the depth of the compressive steel. The maximum value v_m occurs at the centroidal axis. The stress v, which occurs between the neutral axis and the tensile steel, is only 43% of the maximum v_m. It is this stress v which is equal to the diagonal tensile stress and which governs the beam design. The value of v_m is generally of no interest. A dotted line is shown in Fig. 3 to indicate the shear stress that would be obtained from the formula in the 1940 Joint Committee Code[6] (or Eq. 15*a* using $j = \frac{7}{8}$). The stress v is 65% of the value given by the code.

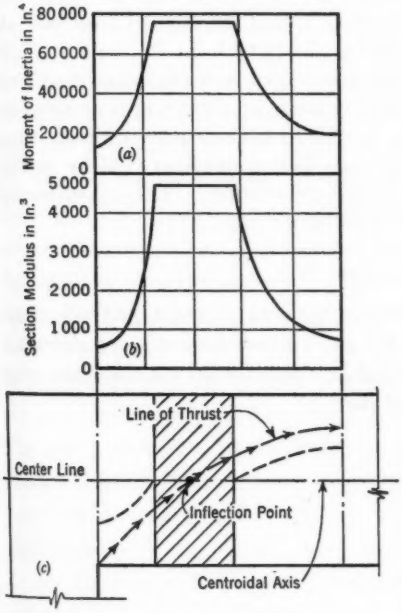

FIG. 5.—SECTION MODULUS AND MOMENT OF INERTIA

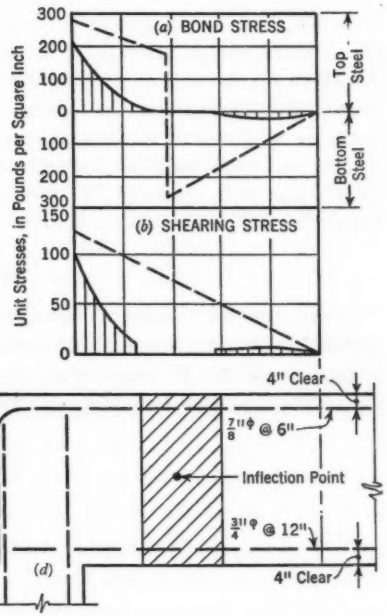

FIG. 6.—BOND AND SHEARING STRESSES

It is of interest to construct bond and shear-stress diagrams along a member. Fig. 4 shows a slice, 1 ft thick, of a rectangular conduit with loads roughly equivalent to the effects of an 80-ft fill. The weight of the conduit has been neglected for convenience. Moments in the frame were computed by moment distribution assuming each member to have infinite rigidity over its supports. Only the clear span of the top slab will be used for discussion. The moment in the top slab at the face of the support is found to be 78.1 ft-kips, the shear is 50 kips, and the thrust is 44 kips. Dividing the moment by the

[6] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 1, p. 52.

thrust gives an eccentricity of the thrust equal to 21.3 in. From this information the neutral axis may be located and the section modulus S_t found to be 535 in.[3] The shear and bond stresses are found from Eqs. 8 to be (using $n = 10$): $v = 94$ lb per sq in. and $u = 204$ lb per sq in., respectively.

Similar analyses were made at various sections of the member which correspond to every 1 in. difference of thrust eccentricity. The section of Fig. 3 was taken at 3 in. from the face of the support in the top slab of the conduit. The locations of the centroidal axis and the line of thrust are shown in Fig. 5(*c*). The variations of the moment of inertia and the section modulus S_t are shown in Figs. 5(*a*) and 5(*b*), respectively. These diagrams illustrate the divergence of a reinforced concrete beam from the ideal straight homogeneous beam which is assumed in the development of flexural theory.

Diagrams of bond and shearing stresses are shown in Fig. 6. The solid lines indicate the stresses as computed by Eqs. 8, and the dashed lines indicate the stresses as computed from the formulas of the 1940 Joint Committee Code. No shearing stress is shown for the shaded region of the beam, since the thrust falls within the kern in this region. There is no diagonal tension in this region.

Stress analysis of reinforced concrete is still in serious need of both theoretical and experimental research to eliminate the unnecessary safety factors which are now inherent in standard design procedures. The experimental research work is needed on actual rigid-frame structures.

Conclusion

Formulas for bond and shearing stresses have been developed which are applicable to members with or without thrust. These formulas are correct in so far as ordinary flexural theory is applicable to reinforced concrete and yield the commonly used formulas as a special case.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

HYDRAULIC MODEL INVESTIGATION OF MITER GATE OPERATION

BY MAURICE N. AMSTER,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The miter gate is a commonly used type of closure for the chambers of navigation locks. It is a two-leaf arrangement, comparatively simple in design and adaptable to convenient means of operation. In the open position the two leaves are recessed in opposite lock walls with their faces parallel to the walls. For closure each leaf is swung about a vertical axis near its downstream (quoin) end so that the upstream (miter) ends of the leaves contact at the center line of the lock. The bottoms and the quoin ends of the leaves are equipped with sealing arrangements that contact floor and lock walls when the leaves reach their closed positions. In supporting a water load, the gate acts as a three-hinged arch, with the two quoin bearings and the miter bearing forming the hinges and the lock walls acting as abutments for the end thrust. The quoin end of each leaf is supported at the bottom by a pintle assembly and at the top by a gudgeon assembly. Frequently used types of operating machines are: Hydraulic cylinder and piston, cable and drum, and crank and strut.

In the design of machinery for the operation of lock miter gates, it is essential to know, as accurately as possible, the power requirements and the loads that will occur on the operating mechanism throughout the cycle. For lack of better data, designers in the past have had to resort to empirical methods usually based upon arbitrary assumptions as to hydraulic resistance. Mechanical difficulties in operation generally have been avoided by the use of large safety factors. In the design of the operating machinery for the gates of the Third Locks of the Panama Canal, the size of the leaves indicated that the forces to be encountered would exceed those in any similar structures in the past. Because of the large number of machines required, the use of high

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 1, 1944.**

[1] Formerly, Asst. Engr., Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

safety factors would have been extremely uneconomical. Accordingly, it was decided to conduct model tests to investigate the resisting forces that might be expected in the operation of the Third Locks gates.

The Prototype

The mechanism designed for operating the prototype gates is of the crank-and-connecting-rod type, having kinematic characteristics similar to those used in the existing Panama Canal Locks but differing in mechanical details

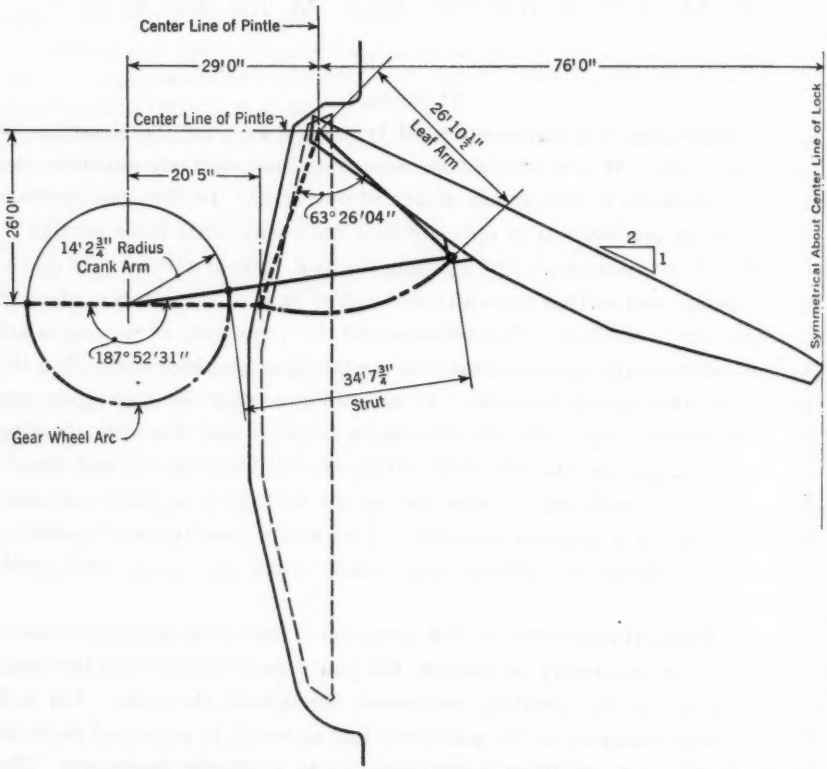

Fig. 1.—Linkage of Operating Machine (Dimensions Are Prototype Values; Scale, 25 : 1)

and arrangement. A large gear wheel, driven by two electric motors through trains of speed-reduction gears, rotates a vertical shaft at practically uniform velocity. A crank is keyed on the lower end of the shaft, and a strut is linked to the end of the crank and to the face of the gate leaf. The operating linkage is shown schematically in Fig. 1. The crank arm is driven through an arc of 187°52′31″ to move the gate leaf through an angle of 63°26′04″. The dead-center positions of the crank relative to the strut coincide with the extreme

positions of the gate leaf, effecting an infinite theoretical mechanical advantage at those positions.

The Model

The model lock, constructed of concrete to a scale of 1 to 25, consisted of a gate bay located in a flume of sufficient length to permit simulation of the lock

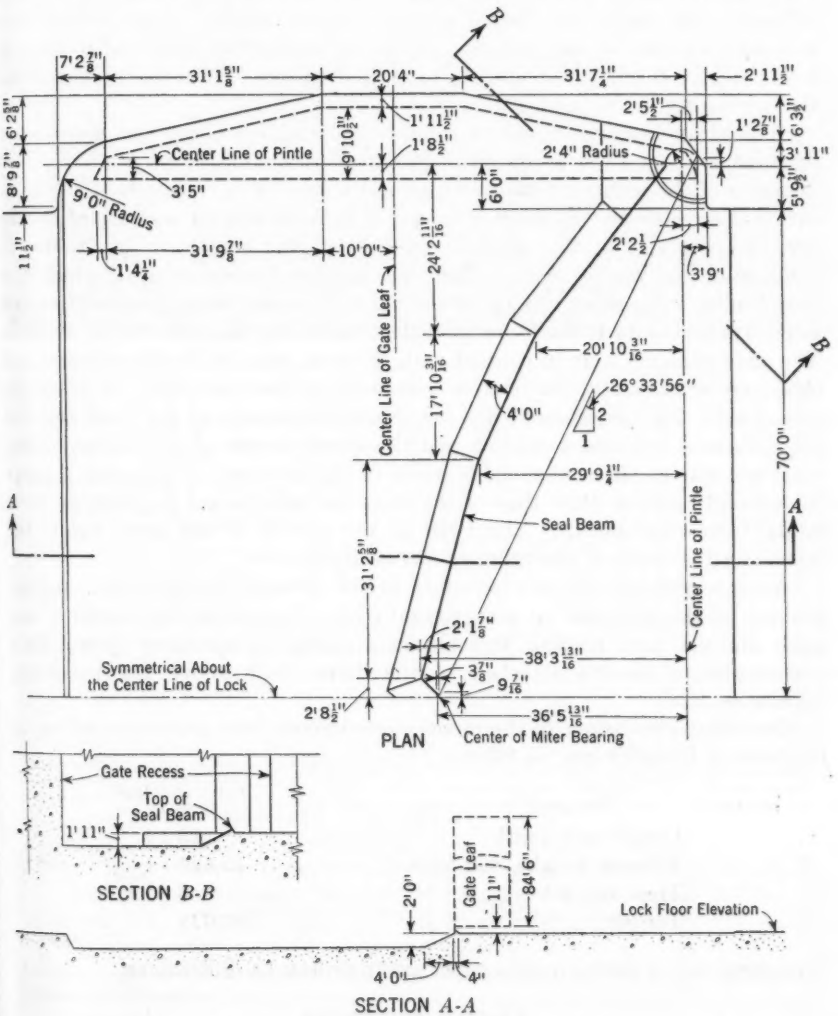

Fig. 2.—Prototype Gate Bay, Recess and Sill

chamber for distances of 1,125 ft upstream and 625 ft downstream. Details of the gate bay are shown in Fig. 2. The gate leaves were constructed of an angle-iron framework faced with galvanized sheet metal. Holes bored in the tops

and the bottoms of the leaves permitted water to flow easily into or out of the leaves during changes of the water-surface elevation in the flume. The weight of the prototype leaves was not simulated.

The model leaves were operated by the mechanism shown in Fig. 3. The position of each linkage relative to its leaf was displaced 180° about the vertical pintle axis for convenience in the model. An electric motor drove two speed reducers with crank arms keyed to their vertical shafts. Each model strut was pin-connected at one end to a crank arm and at the other end to an arm keyed to a vertical alloy-steel shaft $1\frac{1}{4}$ in. in diameter. This shaft transmitted the driving torque to the leaf.

The torque applied to one of the leaves during operation was determined by measuring the twist produced in a reduced section of the drive shaft. The diameter of the reduced section of each shaft was 0.6 in. Details of the measuring apparatus are shown in Fig. 4. A hollow cylinder was keyed to the drive shaft just below the reduced section, and a horizontal arm was bolted to a flange on the top of the cylinder. On another horizontal arm, which was keyed to the drive shaft just above the reduced section, were mounted two dial gages graduated to indicate linear deflections to the thousandth of an inch. The gage plungers were in contact with the arm attached to the cylinder and thus were actuated by the relative movement of the two arms. A value obtained with this arrangement was not exactly a measure of the twist arc, but the difference between a reading and the actual length of the corresponding twist arc was so slight that calibrations of the reduced section over a range considerably greater than that of the tests did not deviate appreciably from straight-line relationship. The twist in the section of the shaft below the point of attachment of the cylinder was negligible.

An extension of the arm attached to the cylinder indicated the angular position of the gate leaf on a segmental scale. Instantaneous values of leaf angle and dial-gage reading were obtained during an operating cycle with a motion-picture camera attached to the drive shaft above the measuring apparatus.

Operating dimensions of the model were derived from prototype values on the basis of Froude's law, as follows

Dimension	Ratio, prototype to model
Length and depth	25 : 1
Volume, weight, and force	15,625 : 1
Time and velocity	5 : 1
Torque	390,625 : 1

Numerical values stated in this paper are in prototype dimensions.

Testing Procedure

The water level in the flume was adjusted to submerge the gate leaves to the desired depth. Fresh water, only, was used in the tests. The operating speed was adjusted by means of the vari-speed pulley arrangement to complete an operating cycle in the required time. It was established by numerous trials

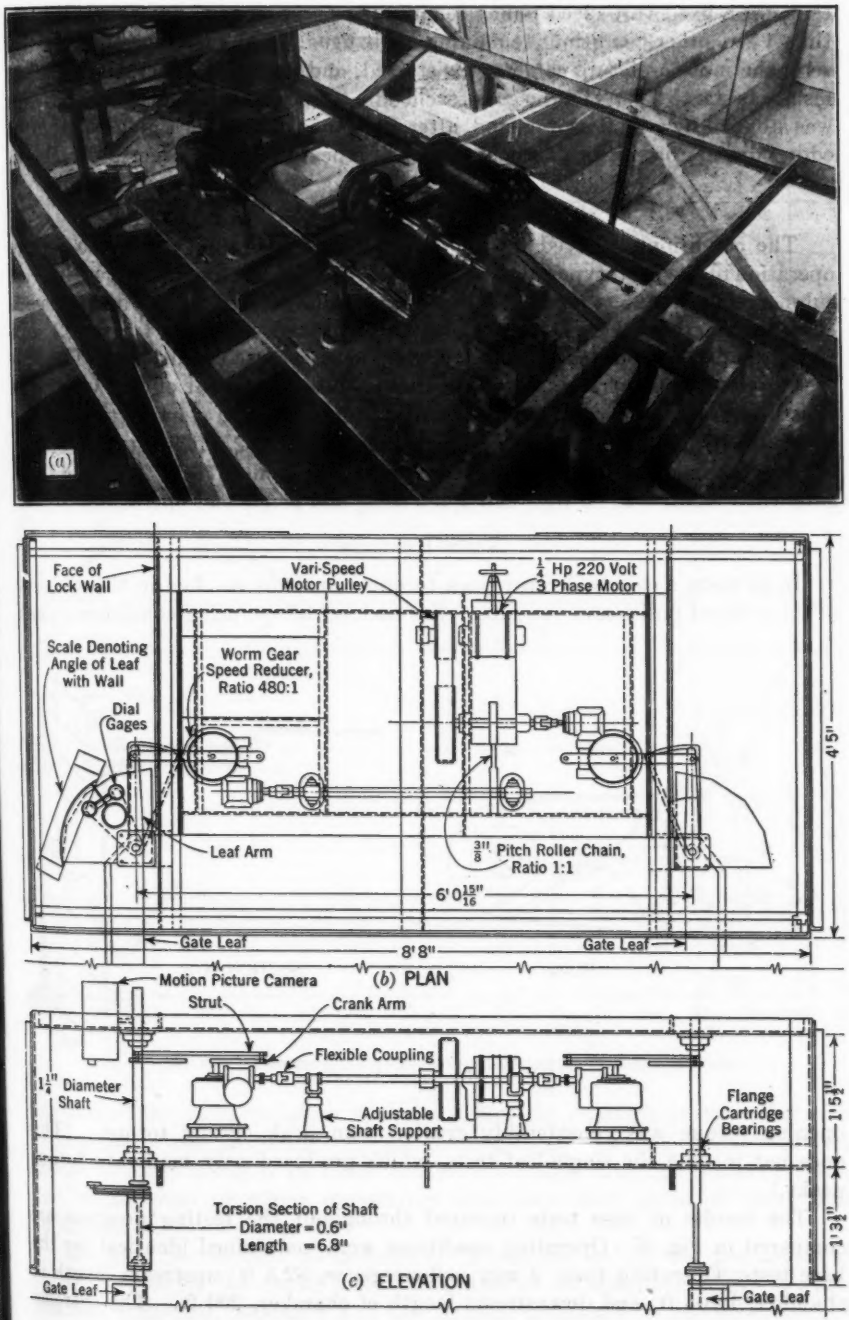

FIG. 3.—MECHANISM FOR OPERATING THE MODEL

that, for a fixed vari-speed pulley setting, the duration of the cycle varied less than 1 sec, other conditions remaining unchanged. The dial gages were set at zero, the motion-picture camera was started, and finally the operating mechanism was started. When the gate reached its terminal position, the mechanism was stopped by a limit switch, and, after allowance of time for the subsidence of eddy currents or surges, the operation was repeated for the reverse direction.

Tests

The conditions believed to involve maximum hydraulic resistance in the operation of the prototype gates were: Synchronous operation of the two leaves, submergence of 82.5 ft, upstream chamber length of 1,105 ft, and downstream chamber length of 200 ft. These, together with an operating time of 2 min (which had been selected tentatively) were used as base conditions. The following conditions were tested to determine their individual effects upon the resisting torque: Nonsynchronous operation (differences in starting time of the two leaves from 0 to 20 sec), submergences from 45 to 82.5 ft, upstream chamber lengths from 300 to 1,105 ft, operating times from 1.75 to 2.75 min, two gate-leaf weights—2,000 tons and 2,400 tons, and single-leaf operation.

Test Results

In all tests, a clearly defined peak torque value was reached in the vicinity of the mitered position of the gate. For each set of operating conditions, peak

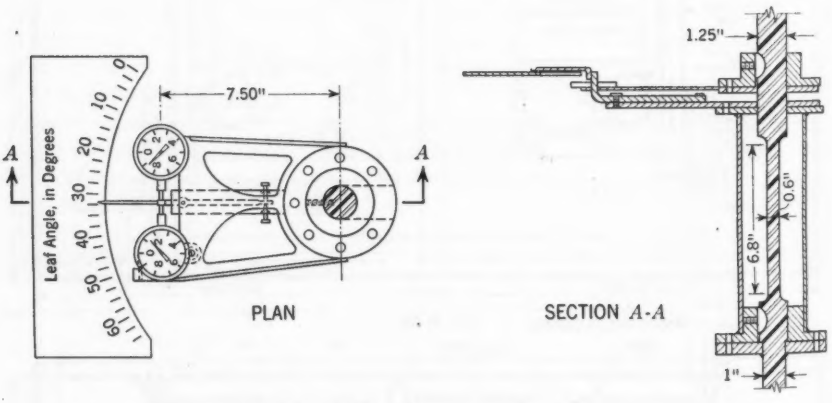

Fig. 4.—Measuring Apparatus

opening torque was considerably greater than peak closing torque. (This does not include the single-leaf tests, which produced only moderate torque peaks.)

The results of base tests repeated throughout the testing program are compared in Fig. 5. Operating conditions were maintained identical for the base tests: Operating time, 2 min; submergence, 82.5 ft; upstream length of chamber, 1,105 ft; and downstream length of chamber, 200 ft. The extreme

deviation from average is about 1,500,000 lb-ft or 8.1% in the opening operation and about 500,000 lb-ft or 3.8% in the closing operation.

The effect of speed variation is shown in Fig. 6(*a*), and the effect of variable submerged depth is shown in Fig. 6(*b*). With the relatively short chamber on

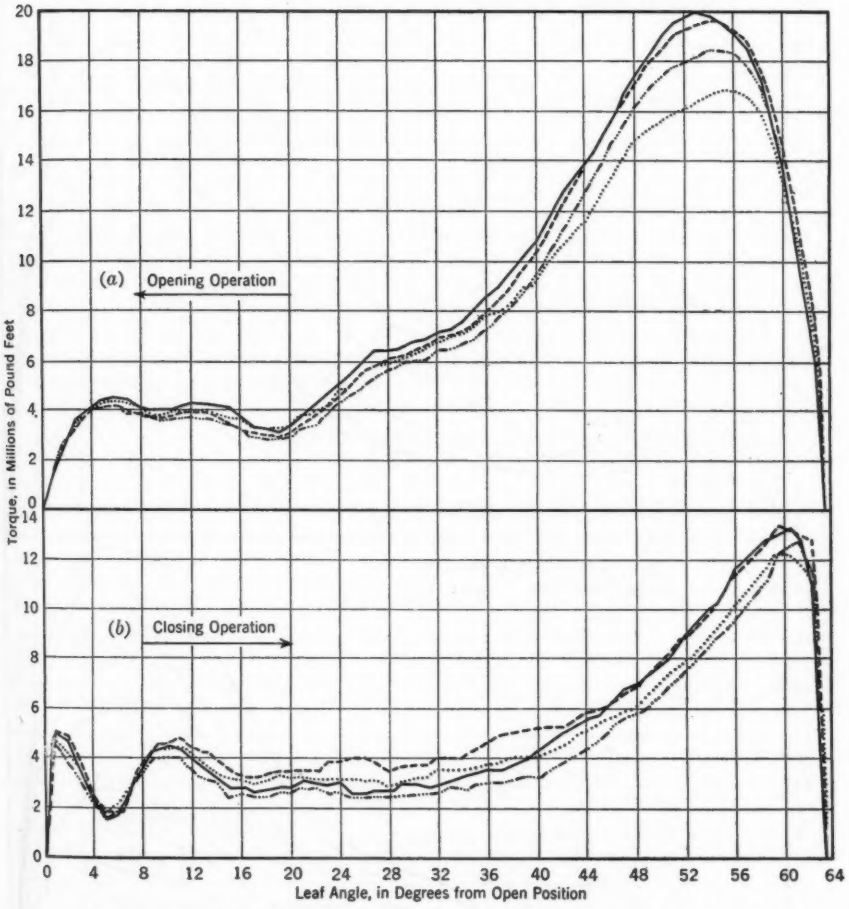

FIG. 5.—MODEL CONSISTENCY, BASE CONDITIONS (OPERATING CONDITIONS CONSTANT)

one side of the gate, variation of the length of the chamber on the other side had no appreciable effect on the torque for lengths greater than 625 ft. Curves, similar to those in Fig. 6, for these variations indicated that the effects of chamber length were of minor importance compared with those of speed and submergence.

Likewise, the effect of an increase of 20% in leaf weight was not sufficient to be revealed by the model tests. The addition of weight increases the mechanical frictional resistance and the inertial resistance to acceleration, but

generally the forces required to overcome bearing friction and inertia are quite small relative to the total resisting force. The angular acceleration of the

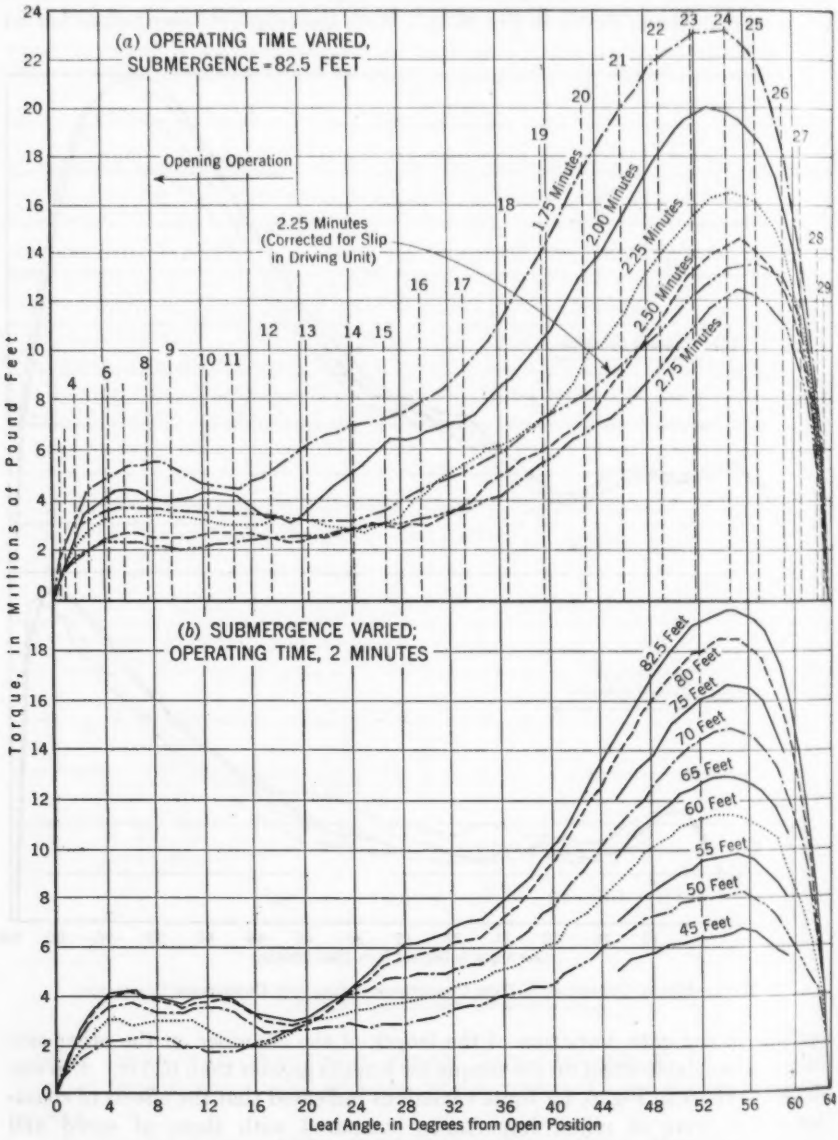

FIG. 6.—EFFECT OF VARIABLE OPERATING CONDITIONS (OPENING OPERATION)

prototype gate leaf was calculated to be about 0.00019 radians per sec per sec at the time of indicated peak torque, and the moment of inertia about the axis of rotation was estimated to be about 208,000,000 slug-ft². Therefore, the

torque necessary to accelerate the leaf at peak-torque position would be 208,000,000 × 0.00019 = 39,500 lb-ft. Since the peak torque values in the opening operation of the base tests ranged from 17,000,000 to 20,000,000 lb-ft (Fig. 5(*a*)), the inertial component is only about 0.2% of the total.

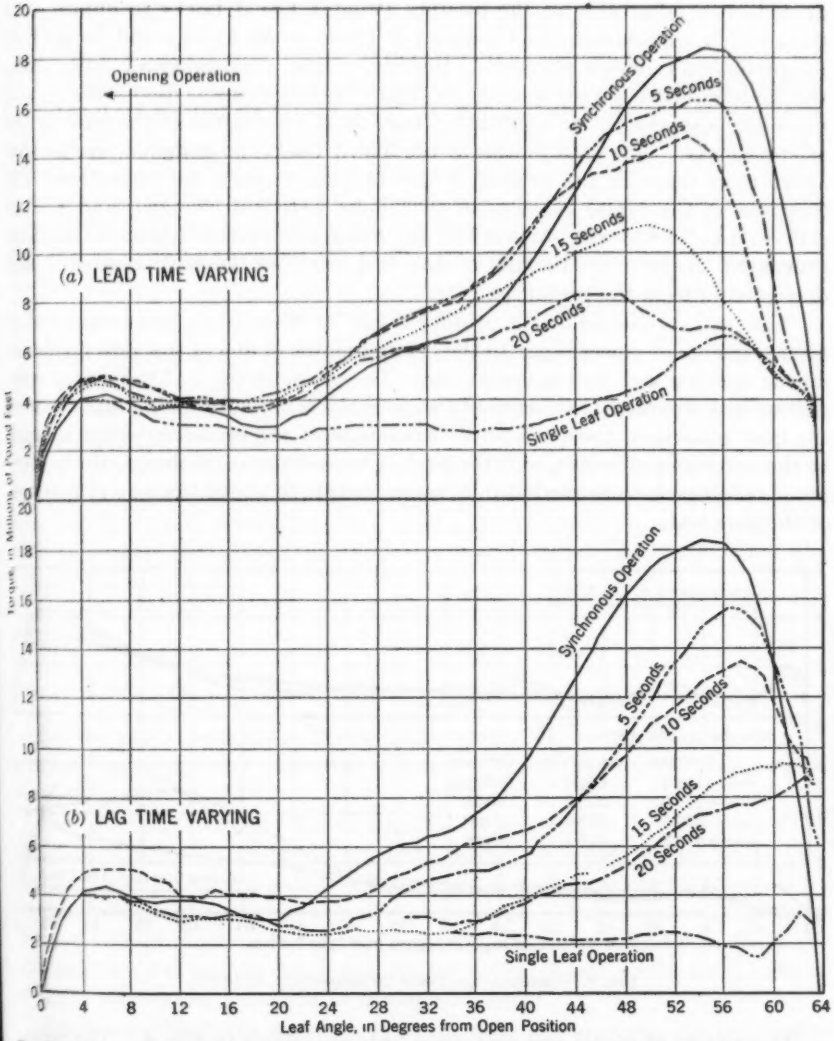

FIG. 7.—EFFECT OF NONSYNCHRONOUS OPERATION OF LEAVES (OPENING OPERATION, BASE CONDITIONS)

Fig. 7 demonstrates that the torque was lowered considerably in the vicinity of the peak values by delaying the starting of one leaf. It is observed that a 20-sec delay reduced the peak values about 50%. Comparison of Figs.

7(*a*) and 7(*b*) indicates that the leaf farthest from the mitered position always required the greater peak of operating torque.

Verification Tests

Prior to the initiation of the model tests just described, tests had been conducted on a miter gate of the existing Panama Canal Locks to obtain data from which approximate load values for Third Locks gates could be derived for preliminary design purposes. Because of the availability of these data, the model constructed for tests on the Third Locks gates was used, with certain necessary alterations, for verification tests on the operation of the gate of the existing locks. The gate pintles in the Third Locks, as designed, are farther apart than those in the existing locks, but to simplify the alterations, the locations of the pintle bearings in the model were not changed, and a scale ratio of 1 to 18.813 was obtained for the verification tests. Space limitations prevented model reproduction of the lock-chamber lengths upstream and downstream from the prototype gate.

The tests conducted on the prototype gate of the existing locks consisted of measurements of power input to the driving motor of one of the gate machines during opening and closing operations. The characteristics of the motor were determined separately by means of Prony-brake tests. By taking into account the gear reduction, the mechanical advantage of the crank-and-strut linkage at the recording intervals, and the probable mechanical efficiency, the power-input readings were converted into values of torque about the axis of rotation of the gate leaf.

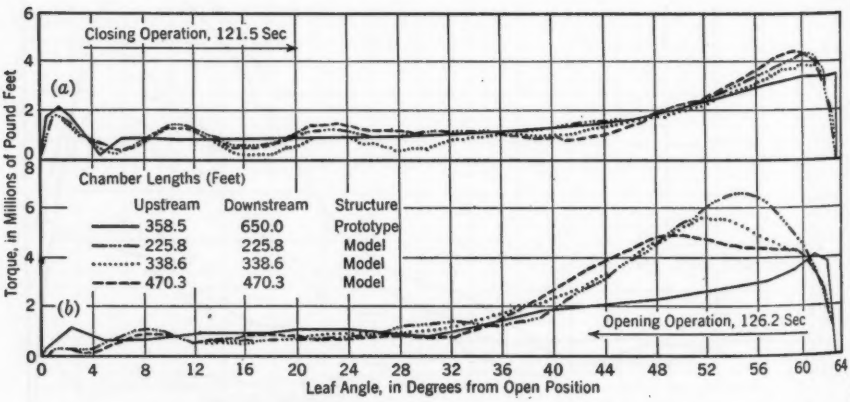

Fig. 8.—Verification Tests (Submergence, 73.16 Ft)

The results of model and prototype tests are shown in Fig. 8. The prototype data represent the average of five test runs, and the model data represent a single run. In all cases, the peak torques observed in the model tests were larger than those recorded in the prototype tests and occurred farther from the mitered position of the gate leaf. The verification is reasonably close for the

closing operation but not for the opening operation. There are several factors that could have contributed to the disparity:

(*a*) The driving motor of the model machine had negligible slip. The slip of the prototype motor was found to be very high at the position of peak resisting torque, and the consequent deceleration would tend to reduce the resisting torque.

(*b*) In the prototype tests, it was observed that the two gate leaves did not start or finish their movements simultaneously, even though the operating machines were started simultaneously. The full importance of this condition, which is probably attributable to a difference in the characteristics of the two motors, was not known at the time of the prototype tests, but its influence was demonstrated by the results of the model tests on the Third Locks gate with unsynchronized operation of the leaves (Fig. 7). Delay in starting a leaf in the opening operation reduces the peak load and delays its occurrence.

Conclusions from Tests

Three conclusions are derived from the foregoing:

1. When the two leaves are operated together, peak resistance occurs in the early part of the opening cycle;
2. The peak load can be reduced considerably by delaying the starting of one leaf, but the best load distribution for simultaneous operation of the leaves, whether synchronized or not, can be obtained by the use of an operating cycle involving low speeds at the extremes of travel—especially near the mitered position; and
3. Peak torque varies directly as operating speed and submerged depth of the gate.

Application of Test Results to the Prototype Design

Factors not represented in the model tests were: Hydraulic drag, mechanical friction, transient surge phenomena, salinity heads, and speed variations in the driving unit.

The hydraulic drag and the mechanical friction forces were assumed to be negligible relative to the total resisting force. Opposing surge heads that might exist at the time a gate is operated and the effect of waves produced by external influences such as wind and ship movement were not considered to be practical of reproduction in the model tests. The determination of the probable magnitude of salinity heads and the correction of test results for the effect of slip in the driving units will be discussed herein.

Notation.—The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[2] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

[2] ASA—Z10.2—1942.

c = distance of the culvert roof from the lock floor (see Fig. 9);
F = force per unit length of gate:
- F_f = force on the fresh-water side of the gate;
- F_n = net unbalanced force;
- F_s = force on the salt-water side of the gate;

T = torque:
- T_a = torque about the axis of rotation;
- T_g = torque at the gear input shaft;

y = depth:
- y_d = downstream, or salt-water depth to the lock floor;
- y_f = depth of water to the equal-pressure plane on the fresh-water side of the gate;
- y_s = depth of water to the equal-pressure plane on the salt-water side of the gate;
- y_u = upstream, or fresh-water depth to the lock floor;

γ = specific weight:
- γ_f = fresh water;
- γ_s = salt water.

Forces Due to Salinity Head.—When the specific gravity of the water on the two sides of a closed lock gate is not the same—a situation encountered at ocean locks supplied with lockage water from a fresh-water source—the attainment of flow equilibrium in the connecting culvert system is very unlikely to balance the total hydrostatic forces on the two sides of the gate. When the fresh or comparatively fresh water is on the upstream side of a closed miter gate, the direction of the unbalanced hydrostatic force is such as to oppose the opening of the gate, as illustrated in the following analysis.

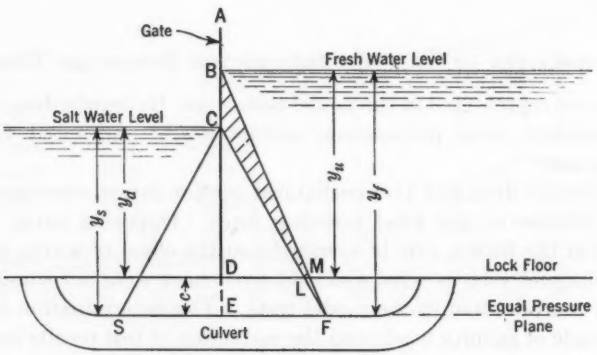

FIG. 9

Fig. 9 is a diagrammatic section parallel to a lock chamber, with the gate represented by the line AD and the pools on the two sides of the gate connected by a culvert running below the level of the gate bottom. For equilibrium to exist and flow to cease, there must be a plane of separation between

salt water and fresh water at which the pressure is balanced. Since salt water is heavier than fresh water, the lower part of the culvert will be filled with salt water at least up to the level of the culvert roof. It will be assumed tentatively that the equal-pressure plane is at the same level as the culvert roof.

The pressure diagrams for the two sides of the gate are represented by lines CS and BF, with $\overline{ES}$ equal to $\overline{EF}$. The area BCLM represents unbalanced pressure on the fresh-water side of the gate.

By definition (see notation):

$$F_f = \frac{\gamma_f y^2_u}{2} \text{ ; and } F_s = \frac{\gamma_s y^2_d}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The net unbalanced force per unit length of gate equals

$$F_n = \frac{\gamma_f y^2_u - \gamma_s y^2_d}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Since $y_u = y_f - c$ and $y_d = y_s - c$ (see Fig. 9), Eq. 2 becomes:

$$F_n = \frac{\gamma_f (y_f - c)^2 - \gamma_s (y_s - c)^2}{2}$$

$$= \frac{\gamma_f y^2_f - 2\gamma_f y_f c + \gamma_f c^2 - \gamma_s y^2_s + 2\gamma_s y_s c - \gamma_s c^2}{2} \quad \ldots\ldots (3)$$

For the condition of equilibrium,

$$\gamma_f y_f = \gamma_s y_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Substituting Eq. 4 in Eq. 3 reduces the force equation to

$$F_n = \frac{\gamma_s y_s (y_f - y_s) - c^2 (\gamma_s - \gamma_f)}{2} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

With the value of y_s known, y_f can be determined from Eq. 4 for assumed values of γ_f and γ_s. With the substitution of $\frac{\gamma_s y_s}{\gamma_f}$ for y_f, Eq. 5 reduces to

$$F_n = \frac{\gamma_s y^2_s (\gamma_s - \gamma_f)}{2\gamma_f} - \frac{c^2 (\gamma_s - \gamma_f)}{2} = \frac{\gamma_s - \gamma_f}{2} \left(\frac{\gamma_s y^2_s}{\gamma_f} - c^2 \right) \quad \ldots\ldots (6)$$

When the connecting culvert is above the level of the gate bottom, Eq. 6 applies without change. If the equal-pressure plane is at the level of the gate bottom, c of Eq. 6 equals 0, and

$$F_n = \frac{\gamma_s y^2_s (\gamma_s - \gamma_f)}{2\gamma_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

In the application of the developed formulas to the prototype, several factors ordinarily would enter into the decision as to which gate would be used as a criterion and into the assumption of salinity values. If no factual data were available, it would be logical to base the computations on the gate that will have the greatest operating submergence and to assume sea water with a

specific weight of 64.0 lb per cu ft on one side and fresh water with a specific weight of 62.5 lb per cu ft on the other side. However, at the existing Panama Canal Locks, which correspond closely with the designed prototype locks and which connect the same bodies of water, information was available as to the location at which the greatest difficulty is encountered from salinity pressures, and the maximum salinity head measured at that location is about 0.7 ft. At the corresponding location in the Third Locks, the maximum salt-water depth to the lock floor on the downstream side of the gate is 68.9 ft, and the lowest culvert roof is 9 ft below the floor. The water on the downstream side is assumed to have a specific weight of 64.0 lb per cu ft. If the salinity of the water on the upstream side were assumed to be uniform for the full depth to the culvert roof, Eq. 6 would apply, but, for a given head difference, the worst condition obviously would be represented by the assumption of sea-water salinity on the upstream side from the lock floor down and a uniform lower salinity from the lock floor up, since pressure difference below the floor does not act on the gate. With this assumption, $y_s = 68.9$ ft and $y_f = 68.9 + 0.7 = 69.6$ ft. Therefore, $\gamma_f = 63.3$ lb per cu ft.

As indicated by this result, entirely fresh water probably will not be found on one side of a gate with undiluted sea water on the other side. Such a condition is prevented by the fact that every time the gate is opened, the sea water on one side displaces a large portion of the fresh water on the other side because of the difference in weight.

The prototype gate leaf is about 84 ft long from miter end to axis of rotation. Neglecting the small area from the axis of rotation to the quoin end and substituting in Eq. 7, the total force on the gate leaf is computed as 141,000 lb. The moment arm of this force about the axis of rotation of the leaf is equal to half the length of the leaf. Therefore, the moment resisting opening of the leaf is $\frac{1}{2} \times 84 \times 141{,}000 = 5{,}920{,}000$ ft-lb.

Correction of Test Results for Slip in Driving Unit.—The model machine was driven by a relatively high-powered motor that produced a practically constant speed for the reduction gear and the operating crank. In the designed prototype machine, however, power is to be transmitted from motors to reduction gears through hydraulic couplings. Therefore the speed variations to be expected in the prototype machine must be taken into account in application of the model test results.

Although most of the model tests were conducted with an operating time representing 2 min in the prototype, the operating time later selected for the prototype was 2.25 min. A tentative selection was made of motors, hydraulic couplings, and reduction gears such that, with the motors operating at rated full-load speed through the entire operating cycle, and taking into account the slip of the couplings, an operating time of approximately 2.25 min would result. The motor and the hydraulic coupling were of standard design, and their slip characteristics were available from manufacturers. For the purpose under discussion, the two slip characteristics were combined. Fig. 10 shows the speed of the output shaft of the hydraulic coupling for the torque range from no load to the breakdown load of the coupling, with the slip of the motor included.

The travel of the gate leaf between recessed and mitered positions was divided for convenience into thirty parts corresponding to thirty equal divisions of the arc of crank movement. The relation of these divisions with respect to angular position of the leaf is shown by the vertical lines in Fig. 6(*a*).

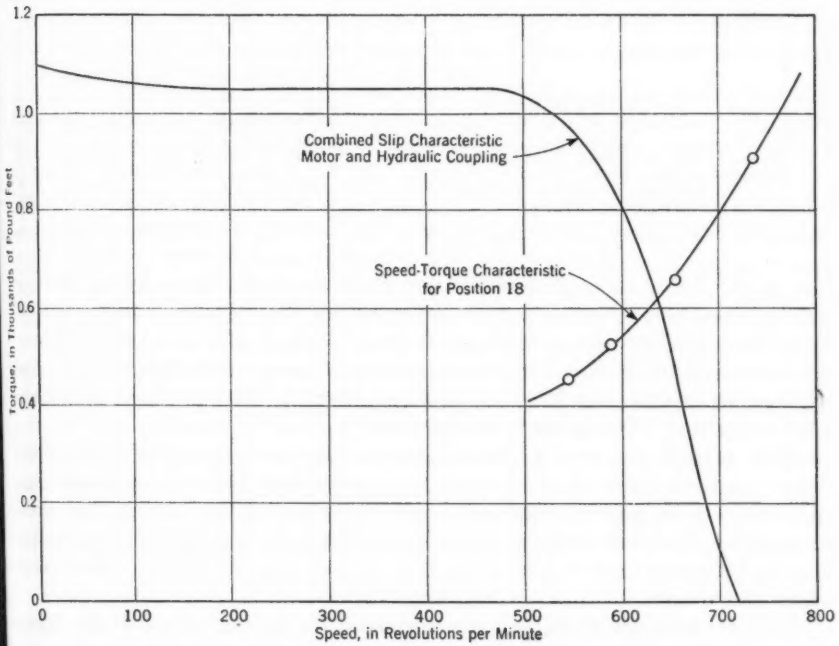

FIG. 10.—CORRECTION OF TEST DATA FOR SLIP IN DRIVING UNIT

Thus, for each position, the resisting torque about the axis of rotation of the leaf is indicated for five different operating times. On the assumption that the crank rotates at uniform speed throughout an operation, each operating time corresponds to a certain speed at the input shaft of the reduction gear. By computing trigonometrically the mechanical advantage of the crank-and-strut linkage for each position, taking into account the total gear reduction, and assuming a mechanical efficiency, the resisting torque at the input shaft of the reduction gear can be derived for each operating speed.

Leaf Position 18 will be taken as an illustration. For this position the mechanical advantage of the crank-and-strut linkage was calculated to be 1.92. The total gear reduction was 5,660 to 1, and the over-all mechanical efficiency, derived from assumed efficiencies for individual parts of the machine, was 89.4%. Therefore, the torque required at the gear input shaft to produce a given torque about the axis of rotation of the leaf was obtained by dividing by the factor $(1.92 \times 5{,}660 \times 0.894 =)$ 9,715. With T_a representing the torque about the axis of rotation and T_g the corresponding required torque at the gear input shaft, the values in Table 1 were derived for the opening movement from

the torque values shown in Fig. 6(*a*) for Position 18. The speed-torque relationships are plotted in Fig. 10 to the same coordinates as the hydraulic-coupling characteristic. Since the output torque of the hydraulic coupling must be the same as the resisting torque at the gear input shaft, the intersection of the curve for each leaf position with the hydraulic-coupling characteristic represents corrected torque and speed for that particular leaf position. Thus the corrected torque at the gear input shaft is 620 lb-ft, and the corrected speed is 637 rpm. The torque at the gear input shaft may be multiplied by the conversion factor 9,715 to obtain a corrected value of torque about the axis of rotation. This gives 6,020,000 lb-ft.

TABLE 1.—Data From Speed-Torque Tests, Opening Movement

No.	Description	Operating Time, in Minutes				
		2.75	2.50	2.25	2.00	1.75
1	Speed at gear input shaft (rpm)	545	589	655	736	818
	Torque, in Pound-Feet:					
2	T_a, about axis of rotation (millions)	4.4	5.1	6.4	8.8	11.7
3	T_g, at the gear input shaft	453	525	659	905	1,205

This method was used to derive torque and speed values for each of the thirty positions for both the opening movement and the closing movement, and, from these values, crank-and-strut loads and motor horsepower were computed. The corrected values of torque about the axis of rotation for the opening movement are shown in Fig. 6(*a*) for comparison with the uncorrected model test results.

To determine the probable operating time, the average speed of the crank between each pair of consecutive positions was assumed to be equal to the average of the corrected speeds at the two positions. The total operating time calculated on this basis differed only a negligible amount from the desired 2.25 min, and the tentatively selected driving unit was adopted for the machine.

The correction method used is an approximate one, of course. The basic assumption was made that the speed-torque relationships indicated by the model test results for any given leaf position would hold true independently of speed variations at other positions. This assumption cannot lead to very accurate results, but the degree of accuracy probably is sufficient for the purpose of the computations.

Capacity of Machines.—All gates of the prototype will be operated by identical machines designed for the operating conditions at the gate with the greatest potential load. This load was assumed to be represented by the test results shown in Fig. 6(*a*). It was unnecessary to make additional allowances for salinity forces; at the gate where salinity is most likely to affect operation, the submergence is only about 70 ft, and, since the machines are designed to operate a gate submerged 82.5 ft, an ample reserve capacity is available (see Fig. 6(*b*)). The hydraulic coupling acts as a load-limiting feature, and, if salinity pressure exceeds expectations, the coupling slip will increase auto-

matically, decelerating the opening movement until the salinity head has been dissipated by flow between the gate leaves. The operating strut is equipped with a shock absorber to dampen the effect of any transient loads such as waves. The shock absorber contains a limit switch that will stop the driving motors before the strut can be overloaded.

Application of Test Results to Other Miter Gates

The data obtained from the model tests probably could be applied to other miter gates having the same kinematic cycle. This would require correlation of scalar ratios with the relationships derived from the tests. The length of a gate leaf would determine the linear scale ratio with respect to the leaf of the test gate, which is 84 ft long. For example, if a gate leaf is 42 ft long, the linear scale would be 1 to 2. If the submergence is 30 ft, the curve in Fig. 6(*b*) representing a 60-ft submergence would be applicable with all torque values divided by 24, or 16. If the operating time is greater or less than 2 min, the appropriate curve of Fig. 6(*a*) is used and corrections are made for a number of leaf positions in accordance with the ratios in Fig. 6(*b*) between the torque values at a 60-ft submergence and those at an 82.5-ft submergence.

If it is necessary to correct for slip in the driving machine, new speed-torque curves should be derived from those of Fig. 6(*a*) on the basis of the scale relationships and the submergence ratio, and the correction method should be applied as outlined herein.

In this application, differences in chamber lengths are neglected. However, unless such differences are great, their effect may be relatively unimportant.

Acknowledgment

At the time of the tests described in this paper the Special Engineering Division of The Panama Canal was directed by T. B. Larkin, M. Am. Soc. C. E., supervising engineer. The investigation was conducted by the Hydraulic Section of the Division under the direction of F. W. Edwards, M. Am. Soc. C. E., senior hydraulic engineer, and later, E. Soucek, Assoc. M. Am. Soc. C. E., senior hydraulic engineer. The model operating mechanism was designed by G. D. Smith, engineer, of the Mechanical-Electrical Section. The writer was in charge of the investigation.

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REPORTS

CLASSIFIED BIBLIOGRAPHY ON THE PHYSICAL AND MECHANICAL PROPERTIES OF WOOD AND THE DESIGN AND CONSTRUCTION OF TIMBER STRUCTURES

PROGRESS REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON TIMBER STRUCTURES

At the Minneapolis, Minn. meeting of the Society in July, 1942, the Structural Division voted to appoint a Committee on Timber Structures "to correlate test data with the design of timber structures especially in regard to the detail of connections." As a constructive first move in this direction, the committee has compiled a list of useful references in this field, classified as to use.

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NOTE.—Please forward all comments on this Report directly to the Secretary of the Structural Division, J. M. Garrelts, Associate Prof., Civ. Eng., Columbia Univ., New York, N. Y. Progress Reports are published in *Proceedings* only.

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Respectfully submitted,

William G. Atwood	Ralph H. Mann
John J. Gould	William A. Oliver
Francis J. Hanrahan	C. H. Sandberg
Verne L. Ketchum	Hermann von Schrenk

H. J. Hansen, *Chairman*
Committee of the Structural Division
on Timber Structures

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

APPLICATION OF SOIL MECHANICS IN DESIGNING BUILDING FOUNDATIONS

Discussion

BY A. CASAGRANDE AND R. E. FADUM

A. CASAGRANDE,[76] M. AM. SOC. C. E., AND R. E. FADUM,[77] JUN. AM. SOC. C. E.[77a]—The discussions, which represent much thought and labor on the part of the contributing engineers, are very stimulating to constructive thinking. Careful study of their contents is recommended to all who are seriously interested in this subject. In addition to presenting a great deal of new material, the discussions contain a divergence of views which should encourage the development of independent thought so important in this field in which design is still largely dependent on judgment.

The discussions have broadened the scope of the paper to such an extent that the authors are not able to comment in this closure on all the new material presented, nor to answer all of the many requests for additional information. In the following, emphasis is placed on material which would not otherwise be available to the readers.

PROPERTIES OF CLAYS AFFECTING SETTLEMENT ANALYSES

The numerous requests in the discussions, for detailed information on the properties of the clays underlying the buildings described in the paper, prompted condensed use of material originally intended for a separate, comprehensive publication on this subject. In this form it also serves as an answer to comments on methods developed by the senior author for analyzing the preconsolidation pressure of clay deposits.

NOTE.—This paper by A. Casagrande, and R. E. Fadum was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Messrs. Jacob Feld, and Leonard Zeevaert; June, 1943, by Messrs. Gregory P. Tschebotarioff, Guthlac Wilson, W. H. McAlpine, and Karl Terzaghi; and September, 1943, by Messrs. D. P. Krynine, O. G. Julian, A. A. Eremin, D. M. Burmister, Abraham Woolf, Lazarus White, George L. Freeman, and Trent R. Dames, Frederick J. Converse, Sterling S. Green, and Paul Baumann.

[76] Associate Prof., Civ. Eng., Graduate School of Civ. Eng., Harvard Univ., Cambridge, Mass.

[77] Asst. Prof., Soil Mechanics, School of Civ. Eng., Purdue Univ., West Lafayette, Ind.

[77a] Received by the Secretary January 11, 1944.

General Description of the Boston Clay Deposits.—The clays underlying Greater Boston are probably among the best explored and most thoroughly tested clays for foundation purposes. They have served as a proving ground for much research in soil mechanics and also for the development of soil sampling methods. In particular, the explorations for the two buildings described in the paper have assisted greatly in improving soil sampling tools and methods. This development culminated in the best sampling of this clay so far accomplished—namely, the samples taken during the explorations for the Mystic Power Station of the Boston Edison Company during 1941–1942. Because considerable trouble in undisturbed sampling was still experienced at the sites of the two buildings referred to in the paper, for which the explorations were conducted from 1936 to 1938, the results of explorations at all three sites will be presented. For the sake of brevity the sites will be referred to as follows:

Site	Building
I	Liberty Mutual Insurance Company Building
II	New England Mutual Life Insurance Company Building
M	Mystic Power Station

At all three sites the soil profiles are similar. Directly overlying rock is glacial till, sand and gravel, deposited and pre-loaded by glaciers. Overlying this material is a thick stratum of inorganic clay, water deposited during the retreat of the last ice sheet. This clay is a fairly continuous formation, but contains many lenses of sand and local pockets of gravel and boulders. Except for the upper hard crust, which at one time was exposed to surface conditions (drying and oxidation), the color of the clay varies between darker and lighter shades of olive green and gray. The term "blue clay" should not be taken literally. This simplified color designation has become part of the name of this clay.

The effect of the drying of the surface has produced the so-called "hard yellow clay" which again is an exaggerated color designation. Beneath this hard clay there is a gradual transition into stiff and medium and finally into soft blue clay.

Because of irregular erosion, the thickness of the hard yellow and stiff blue clay is variable. Over limited areas these materials are entirely absent, and in some places sand has been deposited in their stead.

After this period of drying and erosion, the land subsided again during the past five thousand years or so, bringing the surface to an average El. −20 mean low water (M.L.W.). On top of the clay a deposit of organic silt was formed, averaging 20 ft thick, and this was finally covered by artificial fill as thick as 20 ft. The thickness of the soft clay averages between 50 ft and 100 ft. This clay deposit is chiefly responsible for building settlements in Greater Boston.

The physical characteristics of good undisturbed samples of the blue clay vary between a very brittle material which upon remolding becomes extremely soft and sticky and a material that shows no brittleness, upon remolding softens

much less, and does not become sticky. To distinguish between these extreme types, they will be referred to as "brittle" and "plastic" types, respectively.

The brittle type is easily sampled with thin-wall samplers,[78] the clay practically "melting" under the pressure of the cutting edge. Such samples show no bending of the bedding planes, although improper sampling may result in shear planes. Even rather stiff soils of this type can be sampled with little pressure and can be easily pushed out of the tubing. This type usually has a slightly more greenish color than the plastic type.

The plastic type offers much more resistance to sampling and to the removal of the samples from the tubes, although the samples may be only medium soft. Even with the best sampling procedure in bore holes some bending of the planes of stratification cannot be prevented. This type of clay often contains individual larger grains of sand and here and there a pebble. It is usually less regularly stratified, and sometimes mottled, as if at one time the deposit had been badly disturbed. Also, it is often found adjacent to boulder nests or lenses of sand or gravel.

In addition to these extreme types there are intermediate types which exhibit some brittleness. Since the grain-size distribution and the liquid and plastic limits of all types vary within the same general range, no differentiation is possible by means of such classification tests. The raw material seems to be the same. Differences in intensity of flocculation at the time of deposition (in other words, differences in the structure of the clay) are assumed responsible for the wide variations in physical characteristics. An attempt was made to determine from the contents of Foraminifera whether individual samples are marine or fresh-water deposited. H. C. Stetson, of the Department of Paleontology at Harvard University, Cambridge, Mass., found Foraminifera in all types of the Boston blue clay, but very sparse (only three or four individuals to about 40 g of dry clay), indicating that the deposit might have been laid down in an estuary at some distance from the sea, under brackish conditions.

The distribution of plastic and brittle type clays over Greater Boston shows no regularity. For example, in the Lynn area the highly brittle type, with a natural water content at about the liquid limit, is present at El. −40, whereas at the same elevation in other areas the less brittle types are usually found. (Elevations refer to Boston city base unless they are marked M.L.W. Boston city base is 6 in. below M.L.W.) In the deep borings at site M the plastic and brittle types alternate. At one location in Cambridge the hard yellow clay was distinctly of the brittle type. Yet, in spite of its hardness, it was considerably above the plastic limit and became medium soft upon remolding. From these and other observations it was concluded that, at the same time and with the same raw material, brittle types of clay would appear to have been deposited in some areas and plastic types in others.

At several sites the brittle type predominated near the bottom of the soft clay stratum. This order of occurrence may be responsible for earlier conclusions that the Boston clay at greater depths is extremely soft and not yet fully consolidated. Actually, good undisturbed samples of such brittle clay, from

[78] "Exploration of Soil Conditions and Sampling Operations," by H. A. Mohr, *Soil Mechanics Series No. 21*, Harvard Univ., Cambridge, Mass., 3d Ed., November, 1943.

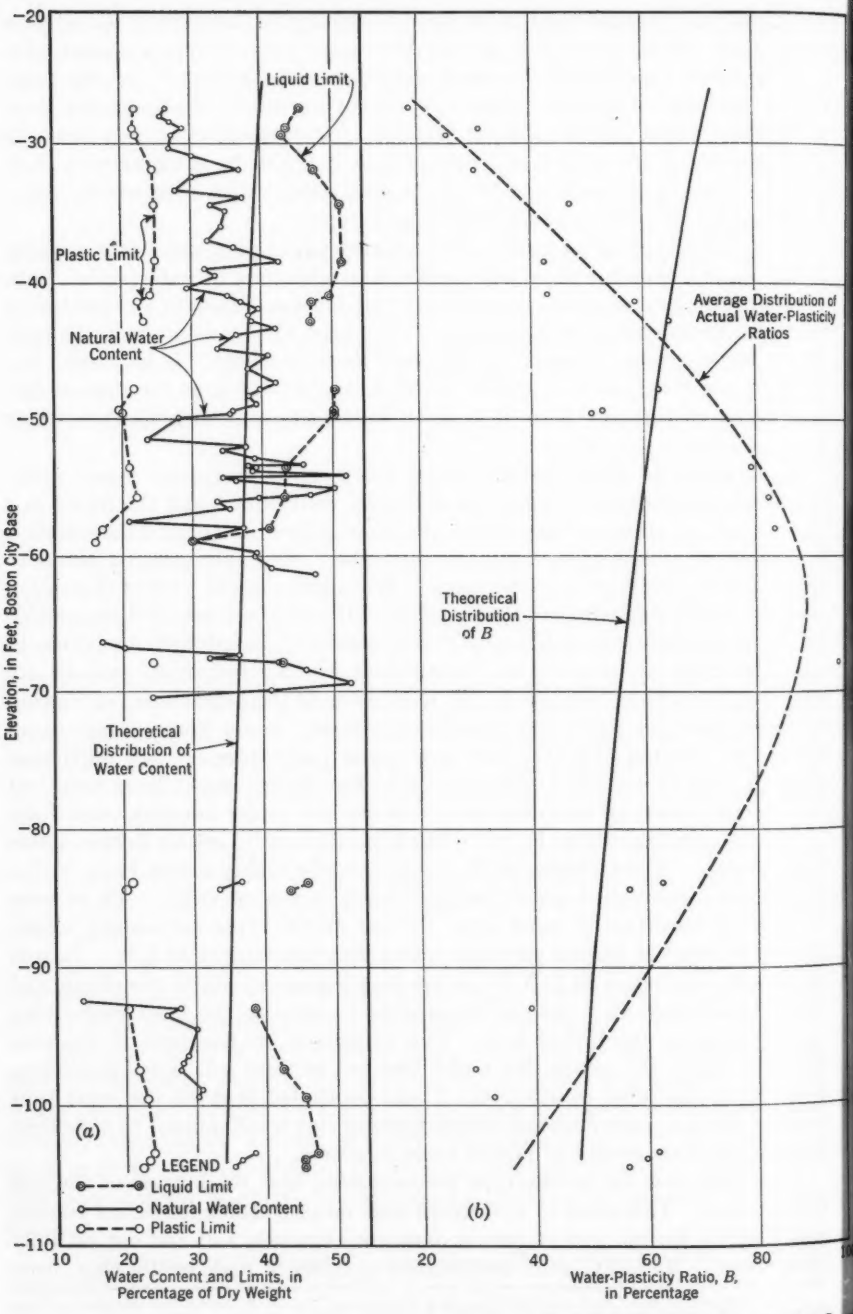
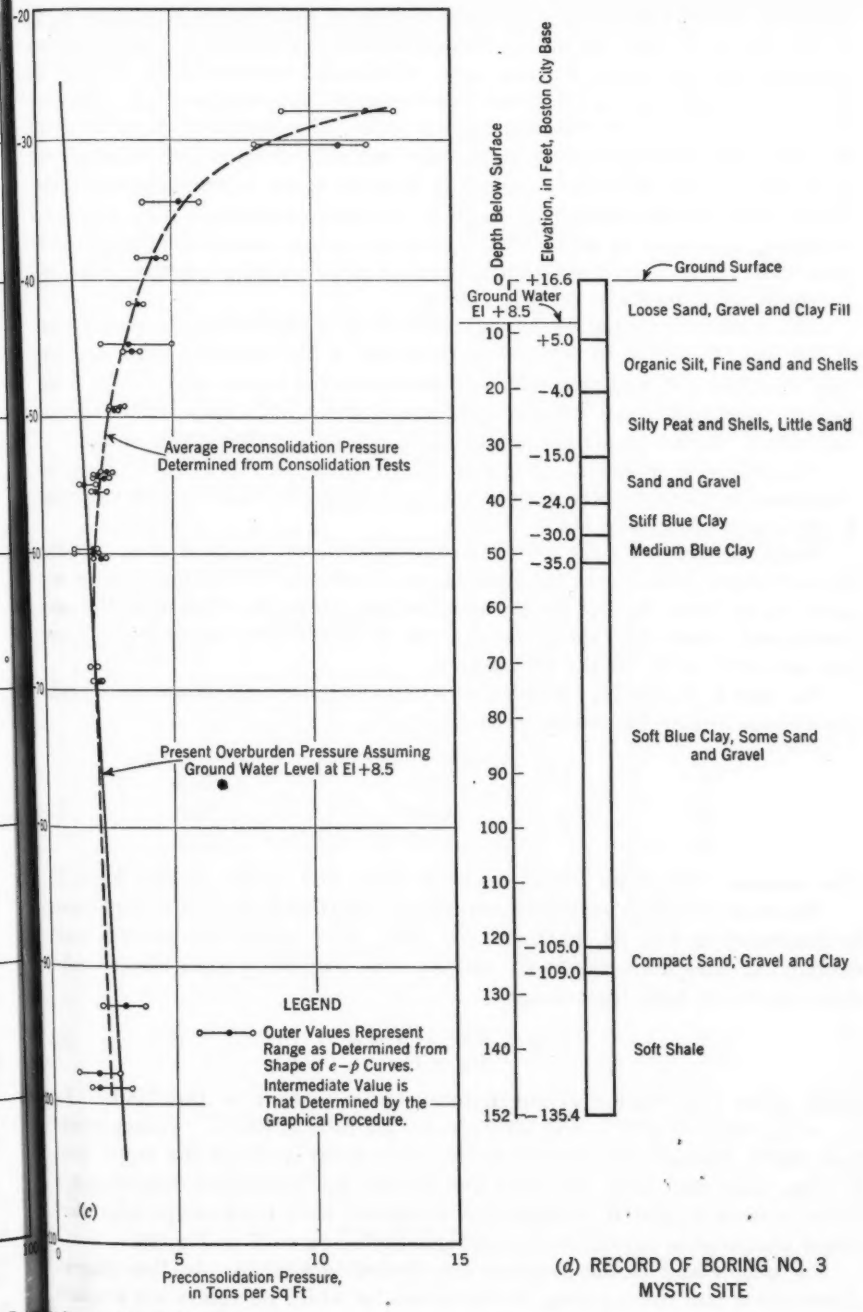


FIG. 33.—PROFILE OF w_n, P_w, L_w, B,



(d) RECORD OF BORING NO. 3 MYSTIC SITE

BORING NO. 3, MYSTIC SITE

depths of about 100 ft, have been found to have compressive strengths up to 2 tons per sq ft; but the use of thick-wall samplers, and the practice of unwatering the bore holes, creating great unbalanced stresses in the clay to be sampled, resulted in such serious disturbance to the samples that, upon recovery, they had to be classified as very soft. This condition, together with the fact that sampling operations become unwieldy and time consuming with great depths, has generally resulted in samples being increasingly more disturbed with greater depth. The use of thin-wall samplers of 2-in. and 3-in. diameter, operating in water-filled, cased bore holes, has eliminated most of these difficulties. Excellent undisturbed samples of brittle clay were obtained at site M from depths as great as 120 ft.

The practice adopted by boring contractors of classifying the consistency of the clay, according to the driving resistance of the sampling tube, as hard, stiff, medium, soft, and very soft, does not reflect the type of clay. The designation "very soft" may actually refer to a medium stiff, very brittle type of clay which "melts" under the concentrated pressure of the cutting edge.

Variations in Water Content and Plasticity in Boston Clay.—The great variations in natural water content can be analyzed by relating water content to the liquid and plastic limits.

From hundreds of limit determinations of samples obtained at many sites in the Greater Boston area the total range of variation for the liquid limit was found to be 30 to 76; and for the plastic limit, 16 to 33. Omitting the very exceptional values, an average range of 40 to 55 is found for the liquid limit; and one of 20 to 25, for the plastic limit.

For sites I, II, and M, for which very detailed soil investigations were made, the average arithmetic means are as follows:

Site	Liquid limit, L_w	Plastic limit, P_w
I	48.3	23.8
II	48.0	22.6
M	45.6	22.3

The over-all average for the liquid limit is 47; and for the plastic limit, 23.

The most complete record yet available in the Boston area for a deep boring is illustrated in Fig. 33, from site M. Fig. 33(*a*) shows the natural water content w_n, and limit values L_w and P_w, and Fig. 33(*b*) gives the so-called water-plasticity ratio (percentage),

$$B = \frac{w_n - P_w}{L_w - P_w} 100 \qquad (17)$$

which gives the relation of the natural water content to the limits. For $w_n = P_w$ this ratio is 0%, and for $w_n = L_w$ the ratio is 100%. Values greater than 100% indicate that the natural water content is above the liquid limit. In Figs. 33(*a*) and 33(*b*), the solid line denotes the theoretical distribution of water content w_n and B, respectively, computed from the average laboratory virgin compression curve, shown subsequently as curve T in Fig. 36.

For sites I and II the B-values are plotted in Fig. 34. In this diagram observations also cover a range in elevations for which no values are available in Fig. 33 because of loss of samples.

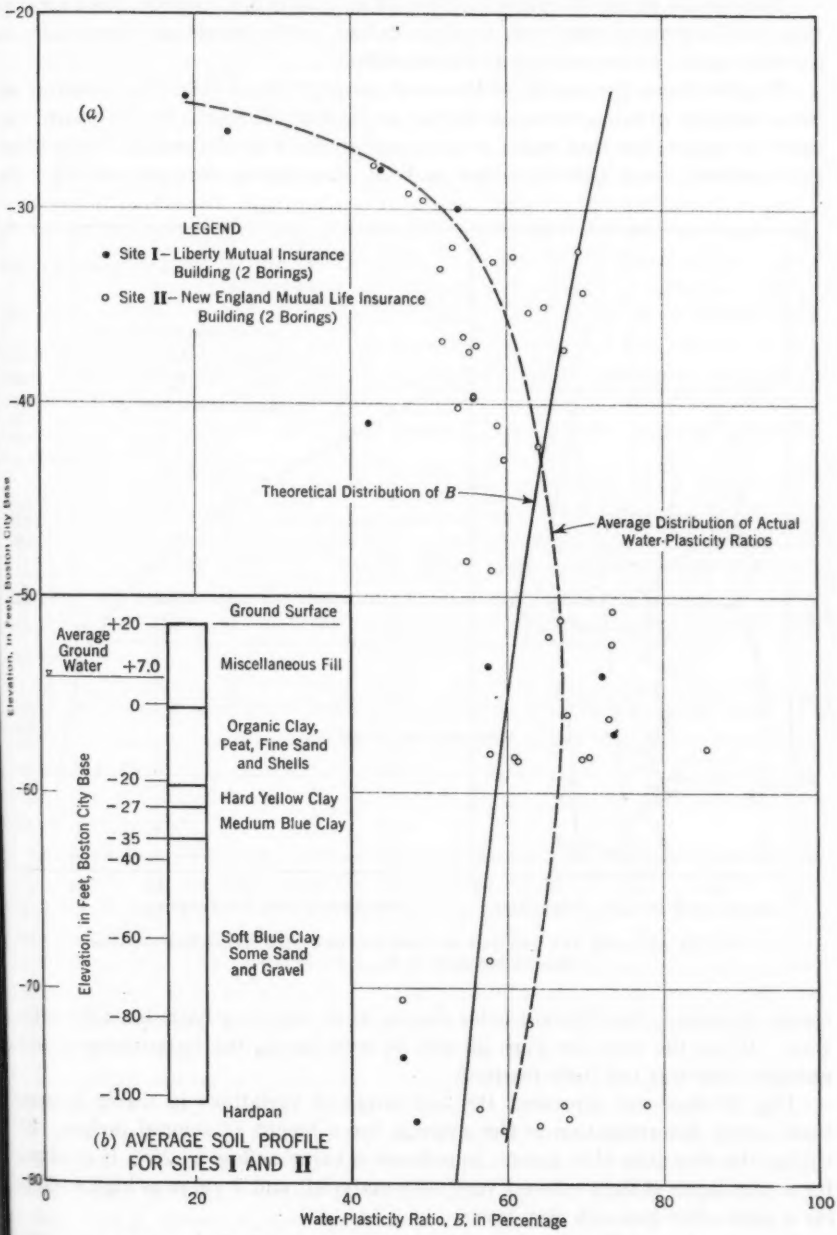


FIG. 34.—WATER-PLASTICITY VALUES FOR BUILDING SITES I AND II

The spread in the B-values in Figs. 33 and 34 is not entirely due to variations in the type of clay but, to some extent, is the result of inaccuracies in correlating the water content and limit values.

Fig. 35 shows the results of the most recent tests of this type, determined from samples obtained from a boring in Boston Harbor. Special care was taken to make sure that water content and limits were determined from identical material, and also that lean and fat clay layers were not mixed. To

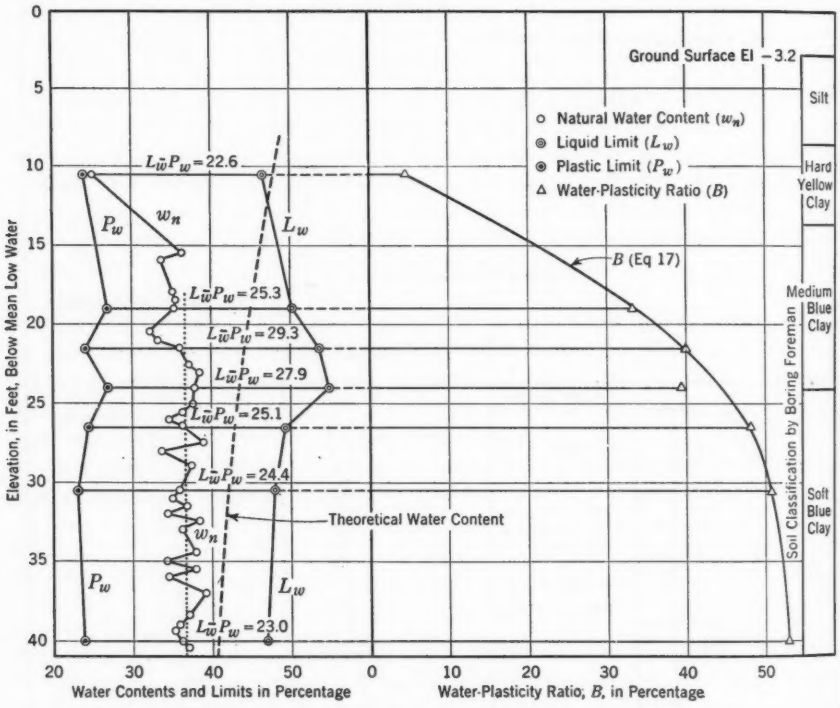


FIG. 35.—BORINGS 14A AND 14B IN PROPOSED DREDGING AREA, EAST BOSTON (GROUND SURFACE AT EL. −3.2 M.L.W.)

check the latter, the different color shades were used as a basis for differentiation. When the tests for Figs. 33 and 34 were made, the importance of such extreme care was not fully realized.

Fig. 35 does not represent the full range of variations in water content, since every determination is the average for a height of several inches. By slicing the clay into thin layers, sometimes a value as low as 25% is obtained for a thin layer of light colored very silty material; and a value as high as 60%, for a dark olive greenish clay layer.

Examination of Fig. 35 shows that the water content does not vary with depth, except for the upper 10 ft to 15 ft. From this, one might conclude that differences in overburden have not consolidated the clay to a water content de-

creasing with depth. If the hard and medium clay had been eroded at this location, as is the case at other sites, one might be even more certain that this clay is consolidated only under the present overburden and shows no difference in water content under widely differing pressures. However, considering that the Boston clay is affected by drying to a thickness of as great as 40 ft beneath the top of the hard clay (that is, beneath the original clay surface before erosion), one expects the water content to increase with depth, rather than to decrease. Actually this increase is noticed only in the hard, and in some of the medium clay. At this profile the plasticity of the clay gradually decreases from El. −23 to El. −40 M.L.W., as evidenced by the plasticity index ($L_w - P_w$) in Fig. 35. Therefore, the same water content at these two elevations actually means that at the upper elevation the clay is more consolidated than at the lower elevation. In this case, only a study of the relation of the water content to the limits—that is, of the water-plasticity ratio—can assure correct interpretation of the meaning of the constancy of the water content with depth. A study of the water content alone would lead to erroneous conclusions.

Water Content of Hard Yellow Clay.—During remolding of an undisturbed sample the water content of the hard yellow clay may appear to be slightly below the plastic limit, because the clay crumbles before it can be molded into a thread. However, the plastic limit of the same sample, after it has been remolded with water for the determination of both limits, is actually slightly below the natural water content, with the corresponding B-value between 0 and 20%.

This discrepancy between the "feel" of the material remolded from an undisturbed state, and the "feel" of that which has been kneaded with the addition of water, has been noticed also for other clays near the plastic limit. Apparently, remolding of hard clay with the addition of water effects a thorough breakdown of the structure, which mere remolding at the natural water content will not accomplish.

Consolidation Tests and Preconsolidation Pressure for Boston Clays.—Fig. 36 shows typical pressure versus void ratio curves for undisturbed samples from the same boring for which other test results are assembled in Fig. 33. The preconsolidation pressure p_p for each curve, determined by the graphical procedure, as well as the probable range of p determined from the shape of the transition curve, is given in Table 1. Generally, the more brittle a sample, the sharper is the transition curve between the recompression and virgin branches, and the narrower is the estimated range for p_p.

The virgin curves are generally straight lines on a semilogarithmic plot, except those for samples with initial void ratios considerably greater than 1.0.

Curves D, E, F, and G, Fig. 36, were obtained from specimens only a few inches apart in elevation. Curve D corresponds to a specimen with a high natural water content, whereas the other three curves represent another type of clay with a water content approximately equal to the over-all average of this boring.

Curve E, Fig. 36, was obtained from a standard test that lasted one week, whereas curve F was obtained from a long-time test that lasted 85 days.

Comparison of the shape of the (e-p)-curves and of the preconsolidation pressures, obtained from them, indicates essentially the same results. These two

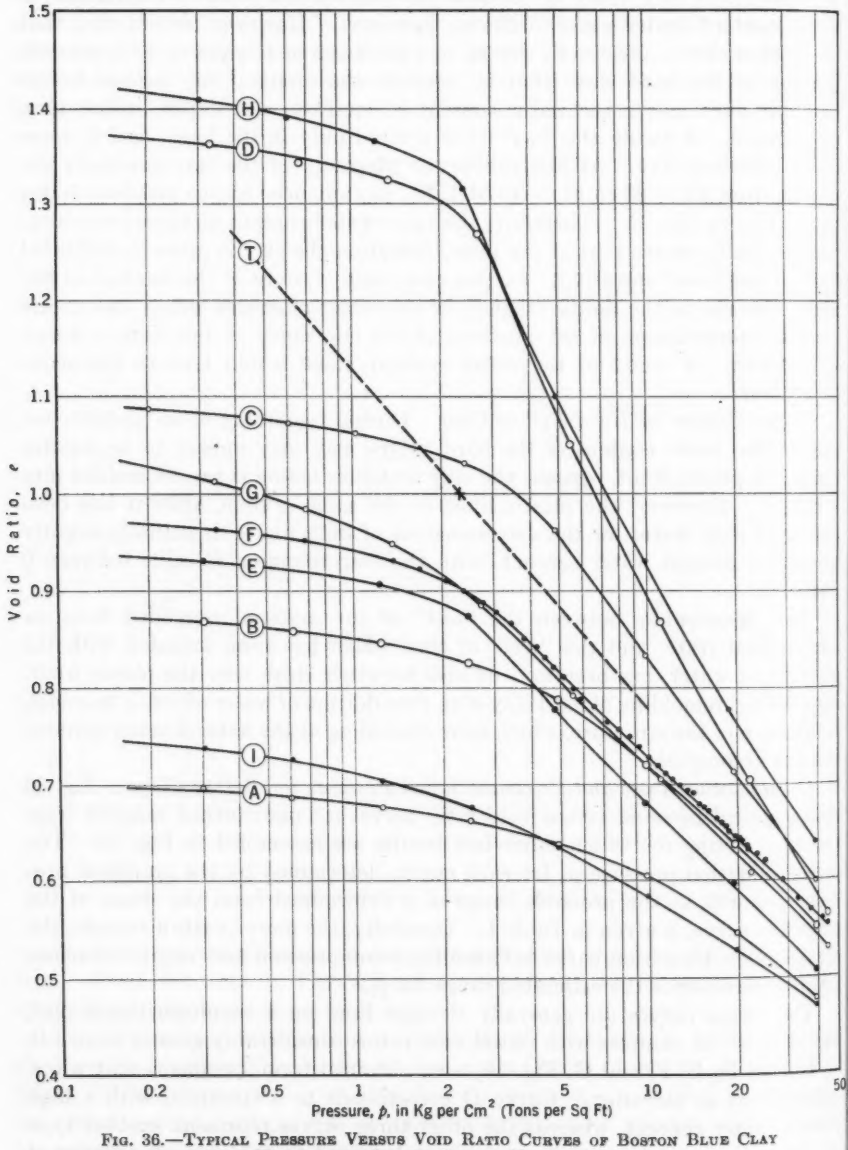


FIG. 36.—TYPICAL PRESSURE VERSUS VOID RATIO CURVES OF BOSTON BLUE CLAY FROM UNDISTURBED SAMPLE BORING NO. 3, MYSTIC SITE

tests, and similar comparisons on other clays, permit the conclusion that consolidation tests lasting several months give practically the same results as the standard tests performed in a few days.

Curve G shows a rather steep recompression branch and a flat transition curve to the virgin branch. This condition is usually caused by excessively distorted samples. For comparison the results are included, showing that the preconsolidation pressures obtained from this test are about 20% to 25% lower than those from the other three tests performed on specimens from the same elevation.

TABLE 1.—PRECONSOLIDATION PRESSURE IN KILOGRAMS PER SQUARE CENTIMETER (TONS PER SQUARE FOOT)

Curve	Laboratory No.	Depth (ft)	Elevation (city base)	Graphical method	Probable Range	
					From:	To:
A	H75-3-4-4A	47.0	−30.4	11	8	12
B	H75-3-5-4A	51.0	−34.4	5.3	4.0	6.5
C	H75-3-7-3B	58.5	−41.9	3.8	3.5	4.0
D	H75-3-10-5A	70.6	−54.0	2.7	2.5	2.9
E	H75-3-10-5B	70.8	−54.2	2.4	2.2	2.5
F	H75-3-10-6A	71.0	−54.4	2.6	2.2	2.8
G	H75-3-11-6B	71.5	−54.9	1.9	1.7	2.3
H	H75-3-12-4B	87.0	−70.4	2.4	2.2	2.5
I	H75-3-14-4	109.5	−92.9	3.3	2.5	4.0
T	Average (e–p)-curve for the computation of the theoretical distribution of w_n and B (see Figs. 33, 34, and 35)					

The preconsolidation values obtained from the twenty-four tests on undisturbed samples from this boring are plotted in Fig. 33(*c*), and for comparison the actual stresses under the present overburden load are shown. From the surface of the clay stratum to about El. −65 the clay appears to be consolidated by drying. Below this elevation there is a good agreement between the p_p-values obtained from the tests and the actual overburden pressure, indicating that the soft clay below about El. −65 may be considered consolidated under the present overburden load. In this case there is fairly clear geological evidence that overburden was never higher than at the present time.

The importance of great care in the performance of consolidation tests, if they are to be used for analyzing the preconsolidation pressure, cannot be overemphasized. Such tests should be supervised and checked carefully, since irregularities in loading, differences in the time allowed for each increment if not adjusted to the same period, etc., easily cause changes in the shape of the (e–p)-curves which may result in large errors in the derived p_p-values. Tests in which some or all increments were not allowed to stand long enough to develop the slope of the secondary compression curve clearly, as indicated by semilogarithmic plots of the time curves (see Fig. 37), should be discarded. Comparison of the consolidation tests for the boring, shown in Fig. 33, with older tests on samples of Boston blue clay obtained with thick-walled samplers indicates that the general magnitude of the preconsolidation pressures, determined by the shape of the (e–p)-curves on less perfect samples, is still of the correct order of magnitude,[79] but that the error in the graphical procedure may increase to 50%.

[79] "Relation of Undisturbed Sampling to Laboratory Testing," by P. C. Rutledge, *Proceedings*, Am. Soc. C. E., November, 1942, p. 1531.

Comparison of Actual and Theoretical w_n-Values and B-Values for Boston Blue Clay.—Fig. 33 also shows the theoretical decrease of water content w_n and water-plasticity ratio B with depth, computed on the basis of an average laboratory compression curve, shown in Fig. 36 as the broken line T ($C_c = 0.355$, through $e = 1.0$, or $w_n = 36.3\%$, and $p = 2.3$ kg per cm²).

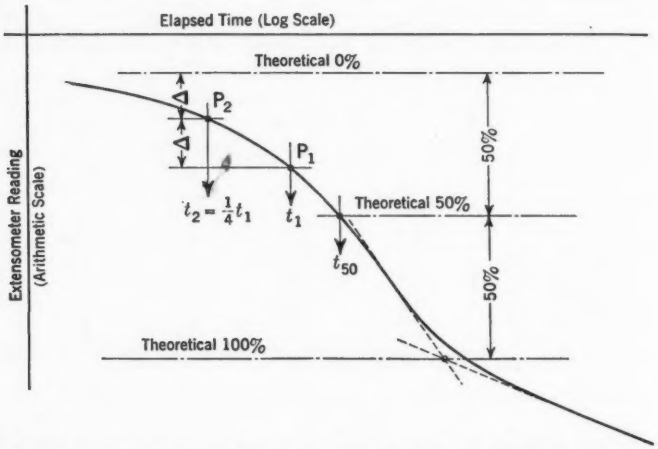


FIG. 37.—ANALYSIS OF CONSOLIDATION CURVE BY MEANS OF A SEMILOGARITHMIC PLOT

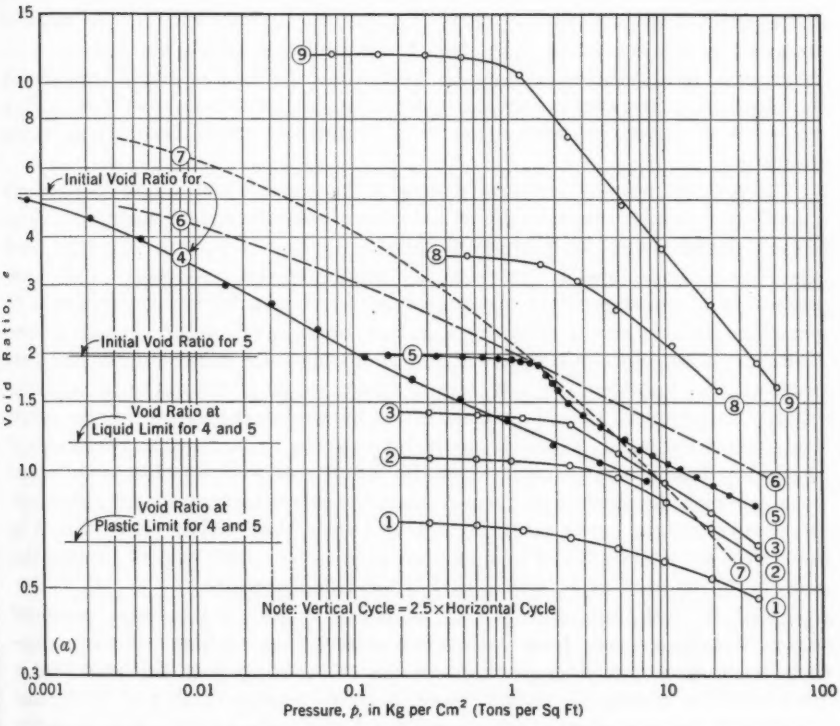
For the zone below the influence of drying, the theoretical decrease in water content from a depth of 80 ft (El. −63.4) to a depth of 120 ft (El. −103.4) is from 36.8% to 34.4%. This decrease is so small that it is completely overshadowed by the actual variations in water content. For the same depths the theoretical B-value (assuming $L_w = 47$ and $P_w = 23$) is reduced from 58% to 48%. The actual decrease in B-value is much greater than the theoretical decrease.

Investigations of Other Clays.—The Laurentian clays permit study of a large range of preconsolidation pressures. The hard Laurentian clays, investigated by Irving B. Crosby, Affiliate Am. Soc. C. E., and the senior author in 1930,[80] are preconsolidated under pressures of from 7 to 15 tons per sq ft, whereas the soft Laurentian clays investigated in recent years by the U. S. Army Engineers are consolidated under pressures of only 1.0 to 1.5 tons per sq ft. Both types of clay are of marine origin and are extremely brittle in the undisturbed state. When remolded, the hard clay becomes very soft and sticky, and the soft clay changes practically into a fluid. Grain-size curves and the Atterberg limits fall in the same ranges. The liquid limits for the hard clay were found to lie between 29.8 and 53.4; and the corresponding plasticity indexes, between 12.5 and 29.8. For the soft clay the liquid limits range between 36.5 and 55.0; and the plasticity indexes, between 12.3 and 28.4.

For the hard clay the water-plasticity ratio B was found to range between 40% and 100%; for the soft clay the range was from 150% to 240%.

[80] Report on an Extensive Investigation of Slides in Eastern Canada, made by A. Casagrande to Irving B. Crosby, Cons. Geologist, January, 1931.

Comparing the theoretical and actual decrease of water content and B-value with depth, the actual decrease is again found to be considerably larger than the theoretical decrease.



(*b*) IDENTITY OF CURVES IN (*a*)

No.	Laboratory No.	Description of material
1	H75-3-14-4	Undisturbed, soft Boston blue clay from site M (see Fig. 36, curve I)
2	H75-3-7-3B	Undisturbed, soft Boston blue clay from site M (see Fig. 36, curve C)
3	H75-3-12-4B	Undisturbed, soft Boston blue clay from site M (see Fig. 36, curve H)
4	H32-12-F2	Flocculated Laurentian clay
5	H32-12-C2	Undisturbed, soft Laurentian clay; Massena, N. Y.
6	....	Hypothetical virgin compression curve for soft Laurentian clay
7	....	Hypothetical virgin compression curve for soft Laurentian clay
8	H29-2-2CP1	Undisturbed, organic silt-clay, Boston
9	H28-1B-CP1	Undisturbed, volcanic clay, Mexico City

FIG. 38.—COMPARISON OF PRESSURE VERSUS VOID RATIO CURVES BY LOGARITHMIC PLOTTING

Such a clay must have been deposited at a water content very much above the liquid limit. Although it is not possible to reproduce in the laboratory the very slow growth of such a deposit, one can arrive at least at a lower limiting void ratio for the condition at which the material was deposited. For this purpose dispersed samples of soft Laurentian clay were permitted to flocculate. Irrespective of the chemical treatment, including one sample which was not treated at all, the deposits were found to have the same void ratio of about 5

after flocculation and sedimentation were completed. Consolidation tests were performed on these samples. The pressure versus void ratio curve of one of these tests (without chemical treatment) is shown in Fig. 38 as curve No. 4. The relationship determined from a consolidation test on the same clay in the undisturbed state is shown by curve No. 5. Thus, even with a void ratio of about 5.0 at the beginning of the test, the flocculated sample yielded an (e–p)-curve considerably below the curve for the undisturbed material. Probably the starting void ratio under the extremely slow natural growth of such a deposit is considerably greater than that obtained by rapid flocculation in the laboratory.

Curves Nos. 6 and 7 are speculations on the possible true virgin (e–p)-curve according to which this clay might have been consolidated by nature. There is some doubt about the meaning of the sharp initial curvature for curve No. 5 after the preconsolidation pressure has been exceeded. Because of the imperfect fit of the sample in the consolidation ring, a lateral strain could be introduced which might result in a partial breakdown of the clay structure. However, three different tests on the same material gave almost identical results. One test was slow (70 days). If the shape of the curve beyond this initial "disturbance" should continue about parallel to the natural virgin curve, one obtains curve No. 6, which could be assumed to begin at a void ratio of about 5. Taking into consideration the much greater drop of w_n-values and B-values with depth than is derived from the laboratory virgin (e–p)-branch, the natural virgin curve must be much steeper than either curve No. 5 or curve No. 6. Curve No. 7 is suggested as a curve which would explain this more rapid decrease of w_n and B with overburden pressure.

Ralph B. Peck, Jun. Am. Soc. C. E., is one of the few who have presented papers [81] containing complete water content and limit profiles. His investigations were made on Chicago clay. When considering only the clay that is unaffected by drying, his data[82] reveal, between depths of 25 and 45 ft, that the natural water content starts nearer the liquid limit at the upper elevation and approaches the plastic limit at the lower elevation. In both profiles the B-value drops from about 70% to about 30%, which is again a much greater drop than the theoretical drop of from about 70% to 60%, computed from laboratory virgin compression curves for similar clays.

Discussion of Variations of w_n, B, and p_p with Depth.—From the foregoing evidence, and similar observations at other localities, it is concluded that, for clays of the types described, the natural water content decreases with depth at a rate at least equal to that which would be expected from laboratory consolidation tests on undisturbed samples. In the foregoing examples the decrease in water content and water-plasticity ratio with depth is actually much greater than the theoretical decrease, which differs from Professor Terzaghi's views on this subject.[83,84] No explanation for this greater decrease is offered,

[81] "Earth Pressure and Shearing Resistance of Plastic Clay," A Symposium, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1008.

[82] *Ibid.*, Figs. 19 and 20, pp. 1009 and 1110.

[83] "Undisturbed Clay Samples and Undisturbed Clays," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., July, 1941.

[84] Discussion by Karl Terzaghi of the paper, "Relation of Undisturbed Sampling to Laboratory Testing," by P. C. Rutledge, *Proceedings*, Am. Soc. C. E., June, 1943, p. 987.

except the tentative suggestion that the magnitude of the secondary compression, which develops over thousands of years, might increase in proportion to the overburden pressure. Thus, differences would result between laboratory and natural (e–p)-curves such as those shown by curve No. 5 and curve No. 7 in Fig. 38, which in turn would cause a greater decrease of water content with depth than the laboratory curve would indicate. However, it should be emphasized that very slow laboratory consolidation tests have so far shown rates of secondary compression along the virgin curve which are independent of pressure.

It is unlikely that the settlements of buildings are governed by (e–p)-curves as steep as curve No. 7 along which nature may have consolidated the clay over long periods. Rather, the (e–p)-curves, obtained from laboratory tests on good samples, seem to form a reliable basis for settlement analyses, provided that the preconsolidation pressure p_p can be determined with sufficient accuracy from laboratory tests, or geological evidence, or both. Without any knowledge of p_p one is forced to assume that the clay is consolidated only under the present overburden. The settlements so computed may be far too large.

The necessity of obtaining as accurate information as possible on the preconsolidation characteristics of clay deposits as a prerequisite for settlement analyses cannot be overemphasized, and the question of how to determine the true p_p is of major importance. Referring to the procedure for determining p_p from consolidation tests, Professor Terzaghi states that he has

> "* * * ceased to give any weight to the numerical results because both the published records and his own experience indicate that the difference between the computed and the real value for samples from the same drill hole may range between zero and several hundred per cent of the computed value. If the maximum preconsolidation load p_o exceeded the present overburden pressure p_1 by less than a few tons per square foot, the graphical method does not indicate whether p_o exceeded p_1 at all. In such instances, the writer relies entirely on geological evidence."

In the experience of the present writers it is just those cases, in which the maximum preconsolidation load exceeds the present load by a few tons per square foot, that require greater accuracy in analysis than is generally provided by geological evidence. The writers believe that this greater accuracy can be furnished by properly conducted consolidation tests on properly taken samples. As to geological evidence, Professor Terzaghi himself states: "* * * but, as a rule, geological evidence regarding the thickness of vanished strata leaves a wide margin for interpretation."

Furthermore, the thickness of vanished strata alone does not determine the preconsolidation pressure of clays. Except for very recent deposits, most clays have experienced one or more periods of drying which geological evidence will usually not reveal. Many apparent disagreements between p_p as determined from reliable tests and from geological evidence are due to preconsolidation of the clay by drying. Such differences can easily be "several hundred per cent of the computed value." In any event geological evidence would not inform the investigator of the degree and depth to which the clay was preconsolidated by drying. Where the telltale, hard, discolored crust has been

eroded, only test results such as those shown in Fig. 33, and those obtained by any other testing methods that may be developed for this purpose, will reveal the existence and extent of consolidation due to drying.

If the large deviations between the true and computed p_p-values are caused by poor samples, or by poorly conducted consolidation tests, the procedure for determining p_p from the test results should not be blamed. Unfortunately, much unreliable information has been derived from tests made on badly distorted samples. Therefore, it is rather treacherous to rely on published data which are several years old, unless one has had an opportunity to check, scrupulously, all the many important details of sampling and testing which may cause erroneous results. It is regrettable that so much data obtained from soil testing of the past, including extensive material in the writers' files, must be eliminated as a source of reliable information because of the use of excessively distorted samples.

Further improvements or development of new methods for analyzing the pre-loading of clays from their stress-deformation characteristics are desirable. The loading history of a clay is a latent picture, somewhat like an undeveloped negative. The success of a settlement analysis depends largely on the ability to develop the picture.

Consolidation of clay by drying affects the magnitude of settlements of buildings in two ways: (1) Reduction in settlements compared to those of a clay of similar thickness that does not contain zones of dried clay; and (2) reinforcement of the clay stratum in the horizontal direction which assists in reducing settlements due to lateral deformation. The latter effect is similar to the reinforcement by layers of coarse-grained soil. Thick flood-plain deposits with their alternating layers of clay, sand, and silt, and with many of their clay layers well precompressed by drying, may show relatively small settlements as compared with the total thickness of clay in such deposits.

The idea of hydrostatic excess pressures in rock underlying clay was advanced by the senior author in 1936 as a tentative explanation for discrepancies in water-content distribution and consistency of so-called undisturbed samples of Boston clay. Observations have indicated that the water pressure in the rock underlying the clay corresponds to elevations above high tide.

It is difficult, if not impossible, to determine the intensity of the true water pressure in the rock underlying clay by water-level measurements in ordinary bore holes. Only by the use of special installations such as those devised for pore-water pressure measurements in clays can the pressure intensity be measured with sufficient accuracy.

Even these measurements must be viewed very critically because of the many technical difficulties involved in eliminating large errors. Not until perfectly reliable methods are developed, will it be possible to prove or disprove for a given site the existence of excess hydrostatic pressures in rock underlying clay.

Beneath extensive impervious clay deposits considerable excess hydrostatic pressure may be a natural and frequent phenomenon that may seriously affect the condition of consolidation of such clay deposits. Any relief in the excess hydrostatic pressure (as produced, for example, by wells) will result in con-

solidation of the clay under its own weight. Large changes in water pressure beneath clay deposits may result in spectacular settlements, such as those observed in the Santa Clara Valley in California.[85]

In this connection, the writers note with interest Mr. White's remarks, revealing a rather emphatic belief in the existence of such pressures.

Logarithmic Versus Semilogarithmic Plots of Pressure Versus Void Ratio Curves.—The remarks in the discussion by the Soil Mechanics Committee of the Los Angeles Section on the logarithmic plot of pressure versus void ratio curves touch a question which has been debated for a number of years.

In 1931 the senior author started plotting the (e–p)-curves of consolidation tests on undisturbed clay samples on semilogarithmic graph paper, chiefly because he found that it facilitated the estimation of preconsolidation pressures. As the results of tests on undisturbed samples on different clays accumulated, he noticed that the virgin compression branch plotted as a straight line for most of these clays. Only for clays with a natural void ratio considerably greater than 1.0, and with tests carried to large pressures, did he find that the virgin branch was more or less curved.

Shortly thereafter, investigators in Europe started independently to use the semilogarithmic plot as well as the logarithmic plot for presentation of (e–p)-curves.

Within the narrow working range of pressures which apply to the settlement analysis of a given structure, the virgin compression curves are practically straight lines, both on semilogarithmic and logarithmic plots. Then the choice may depend on whether the mathematical expression of a logarithmic function, or of a power function, is easier to use in a particular problem. The writers consider the logarithmic equation more easily handled in settlement analyses.[7]

The logarithmic plot has the advantage over the semilogarithmic plot in that it facilitates assembling on one diagram the results of soils with widely different compressibilities, as illustrated in Fig. 38. A number of sedimentary soils, which are curved on this plot, show straight-line virgin curves on a semilogarithmic plot. On the other hand, the clay from Mexico City, Mexico, which is not an ordinary clay but the result of chemical decomposition of volcanic ash, is a straight line on the logarithmic plot.

A minor disadvantage of the logarithmic plot is the necessity of using a much larger cycle size for the void-ratio scale than for the pressure scale to enable sufficiently accurate plotting. When plotting curves on symmetrical log paper, the (e–p)-curves of ordinary clays are so extremely flat that they would always give the impression of being straight lines.

Analysis of Time Curves from Consolidation Tests by Means of Semilogarithmic Plots.—Professor Krynine requested further information on this method. It consists of three parts: (1) Determination of the "theoretical zero," (2) determination of the boundary between primary and secondary compression, and (3) computation of the coefficient of permeability.

[85] "Ground-Water, Salt-Water Infiltrations, and Ground Surface Recession in Santa Clara Valley, California," by C. F. Tolman and J. F. Poland, *Transactions*, Am. Geophysical Union, 1940, Pt. I, pp. 23–35.

[7] "Compression Characteristics of Clays and Application to Settlement Analysis," by P. C. Rutledge, a thesis presented to the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in June, 1939, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering.

The necessity for finding a "theoretical zero" was realized by the senior author in 1928, after he discovered that the air content in soils invalidates the original procedure developed by Professor Terzaghi for analyzing time curves.[86] The method for finding the theoretical zero is based on the assumption that the upper half of the time curve is approximately a parabola, and consists of the following steps (on arithmetic and semilogarithmic plots): Choose an arbitrary point P_1 on the time curve, within the estimated first 50% consolidation. Divide the time, t_1, of this point by 4 and find the corresponding point P_2 on the curve. The difference in ordinates, Δ, of these two points is transferred above P_2, as illustrated in Fig. 37. This procedure is repeated for several points and the horizontal line passing through the average of the resulting locations is assumed to be the "theoretical zero coordinate."

With this modification, the senior author continued using the Terzaghi procedure, but in 1930 discovered very large discrepancies between the coefficient of permeability computed from the time curves and the directly measured values for undisturbed clays. After considerable experimentation with various types of plots, plotting the time on a log scale was found to permit visual differentiation between the primary and secondary compression. Also the intersection of the tangent to the inflection point and of the straight-line continuation of the secondary compression branch was found to correspond approximately to a theoretical consolidation of 100%.

Since both the theoretical zero point and the theoretical 100% point are subject to errors, the theoretical 50% point is considered least affected by these errors and was chosen instead of the 90% point in Professor Terzaghi's original procedure. Then for the time, t_{50}, corresponding to this theoretical 50% point, the coefficient of permeability can be computed from the following formula, based on the Terzaghi theory of consolidation:[87]

$$k = \frac{a_v \gamma_w H^2}{20 (1 + e_m) t_{50}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

in which $a_v = \frac{p_2 - p_1}{e_1 - e_2}$ = coefficient of compressibility obtained from $(e\text{–}p)$-curve for increment; γ_w = unit weight of water; H = thickness of layer drained on both sides; $e_m = \frac{e_1 + e_2}{2}$ = average void ratio for the increment; and t_{50} = time for theoretical 50% consolidation, from Fig. 37.

Using the semilogarithmic plot for time curves, the writers have always been able to check, with satisfactory accuracy, the results of direct permeability tests with the k-values computed from the time curves, provided that the slope of the secondary compression branch was not so steep as to obliterate the bend in the curve indicating the transition from primary to secondary compression. This obliteration happens frequently in the case of peaty soils, but in inorganic clays only when very small load increments are used. The latter reaction is due to the fact that the slope of the secondary compression branch on a semilogarithmic plot is independent of the magnitude of the load increment; in

[86] "Principles of Final Soil Classification," by K. Terzaghi, *Public Roads*, Vol. 8, 1927, p. 46.
[87] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., 1943, p. 274.

other words, the greater the load increment, the sharper the line of demarcation between primary and secondary compression.

The use of the square root scale for the time curve, as introduced by D. W. Taylor, Assoc. M. Am. Soc. C. E., assists in determining the theoretical zero coordinate, but does not eliminate the difficulty of distinguishing between primary and secondary compression. The large discrepancies between the k-values computed from arithmetic or square root plots and the directly measured values are due to this difficulty.

In addition to more accurate computation of the coefficient of permeability, the semilogarithmic plot has an advantage over other plots in that the shape of the theoretical curve remains unchanged regardless of changes in the variables in Eq. 18. Once one is acquainted with the characteristic shape of the theoretical curve, a glance at an observed curve on a semilogarithmic plot will reveal numerous important details regarding the primary and secondary compression that cannot be seen in the other plots. For example, it will show whether the time allowed for consolidation has been sufficient, will furnish an estimate of the degree of accuracy of the computed k-values, or will show whether the secondary compression obliterates the primary compression to such an extent that it is not possible to compute k.

When performing consolidation tests one should always plot the time curves on semilogarithmic paper as consolidation progresses, to enable proper control of the test. Without such a precaution many tests are ruined by allowing insufficient time for consolidation.

Stress Distribution in Clay and Hooke's Law

The omission of stress diagrams showing the results of computations of stress distribution in the clay beneath the buildings was not intended to convey the impression, gained by several discussers, that the writers consider this subject of no importance. In the settlement analyses of the two buildings described, extensive computations were made to determine the stress distribution in the clay for different designs.

A number of papers in *Proceedings* in the past few years have described methods for making these computations. Since the results from all of these methods are sufficiently accurate for practical purposes, the choice of method is a matter of getting results with the minimum effort. For the analysis of building II, the junior author developed a set of influence charts, both for isotropic and anisotropic soils, which proved useful.[88, 89]

The distribution of stress in clay is probably more affected by anisotropic properties of such deposits than by deviations from Hooke's law. In the design of building II, various assumptions of anisotropy were made.[90]

Mr. Julian has presented a clear and scholarly review of those aspects of the theory of elasticity involved in the computation of stresses and settlements.

[88] "Observations and Analysis of Building Settlements in Boston," by R. E. Fadum, a thesis submitted to the Faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering, pp. 265–294.

[89] One of these influence charts is reproduced in "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 485.

[90] "A Problem of Elasticity Suggested by a Problem in Soil Mechanics: Soft Material Reinforced by Numerous Strong Horizontal Sheets," by H. M. Westergaard, *Contributions to the Mechanics of Solids*, Stephen Timoshenko 60th Anniversary Volume, 1938.

His discussion reveals several interesting suggestions that should be of assistance to those desiring more information on this subject. Poisson's ratio of a given clay is not a constant quantity. For the purpose of computing the initial stress distribution clay is usually assumed to be a perfectly incompressible material.

The discussion of the Soil Mechanics Committee of the Los Angeles Section contains two conflicting statements on the question of the validity of Hooke's law. On the one hand, the committee proposes to expand the writers' statements on the limitations of the validity of Hooke's law to various types of soils, as follows: "Hooke's law does not apply to any soil." On the other hand, it proposes that shear stresses under rigid footings should be computed by means of the theory of elasticity, which presupposes the validity of Hooke's law. In addition, the shear stresses as computed for a uniform loading on a perfectly flexible footing are assumed to hold also for rigid footings. According to the theory of elasticity, infinite concentrations of shear stresses will occur under a rigid footing and a statement that the maximum shear stress will be equal to p/π is an unwarranted assumption. Either the material is almost perfectly elastic, in which case the maximum shear stress will be much greater than p/π; or the material is capable of making plastic readjustments where stresses are high, and then the maximum shear stress may be smaller than p/π. Fortunately, most soils have the property of distributing stresses from such parts where the shear strength is approached, to adjacent parts of the soil where shear stresses are moderate. This action can be compared with the manner in which steel adjusts stress concentrations which occur in almost every steel structure. Just as the structural engineer relies on this property of steel, so must the foundation engineer rely on the capability of soils to release high stress concentrations.

Properties of Cohesionless Soils Affecting Settlements and Bearing Capacity

Have Simplified Rules a Place in Foundation Engineering?—In his contacts with the profession, the senior author has received frequent requests for a simplified presentation of the results of soil mechanics to serve as a guide to engineers who have neither the time nor the opportunity for a detailed study of soil mechanics. It cannot be denied that even today many architects and engineers are guided by the misleading concept that each soil has a constant bearing capacity, independent of the size and depth of the loaded area, and of the type of structure (see most handbooks and building codes). The designers are not entirely to blame if many building foundations are still designed by such completely inadequate, although age-old, rules.

Simplified formulations of the findings of soil mechanics have an important place in the design of building foundations and are practicable for coarse-grained soils. These formulations should convey at least qualitative information on the influence of possible variations in the design—for example, width, rigidity, and depth of loaded areas for the pertinent range in which such quantities ordinarily vary in building foundations. However, settlement analysis of buildings on soft soil does not as yet seem ready for such simplification.

In the first part of the paper a number of simplified formulations are presented concerning the behavior of loaded areas on cohesionless soils. These statements were included in an attempt to provide information which might be useful chiefly as a qualitative guide to the designer. Valuable comments were contributed on several of these statements.

(*a*) Discussers object to assuming zero pressure along the edge for rigid, loaded areas on the surface of cohesionless soil because no measurements by pressure cells have ever indicated zero pressure. It would require extremely small pressure cells to measure the pressure along the edge; but extrapolation from measurements made in the interior of such loaded areas has indicated practically zero pressures along the edge. In addition, from the strength characteristics of a cohesionless soil it follows logically that for the zero confining pressure at the edge the other stresses at this point must also disappear.

It was also objected that building footings are always extended to a certain depth below adjacent ground surface. However, there is a definite trend toward placing foundations for smaller buildings directly on the surface of a sand or gravel fill. This is also true of foundations for tanks. Therefore, a designer should know that zero edge pressure is a possible extreme limiting condition which he may have to consider.

In support of his claims that "There is no justification for the assumption that the intensity of pressure at the edge of a rigid area is zero," Mr. Feld refers to tests by Professor Converse.[18] These tests were not performed on the surface but at a depth of 4 ft. Furthermore, the soil was not cohesionless, but is described as "* * * a clay-loam, according to the classification of the U. S. Bureau of Public Roads, and was made up of 29 per cent clay, 30 per cent silt, and 41 per cent sand." [91]

(*b*) In regard to pressure distribution on a rigid area supported at great depth, Mr. Feld states: "The uniformity of base pressure in pier foundations is the result of the rigidity of a deep footing rather than of the depth of the footing." Rigidity only insures uniform settlement. In the case of cohesionless materials, there is adequate proof that deformation does not occur in accordance with Hooke's law; therefore a uniform settlement does not insure a uniform base pressure at any depth.

The comments of Messrs. Terzaghi, Krynine, and Freeman on statement (*b*) are pertinent. Construction procedure, disturbance of the soil, and flexibility of the contact surface will result in considerable deviation from a uniform contact pressure. In most cases it is not possible to evaluate the influence of all these factors, and the assumption of a uniform contact pressure usually will be found to be on the safe side.

(*c*) Mr. Feld suggests rewording the statement on increase of ultimate bearing capacity with width of footing "* * * to the effect that the unit bearing is independent of the width of the loaded area." This is just the opposite of the writers' statement.

Professor Terzaghi states that the increase of ultimate bearing capacity approximately in direct proportion to the width of the loaded area "* * * does

[18] *Civil Engineering*, April, 1933, p. 207.
[91] *Ibid.*, p. 208.

not apply to sands whose angle of internal friction decreases appreciably with increasing pressure." In the writers' experience, the angle of internal friction for a given sand can be assumed substantially constant in the range of pressure intensities encountered in normal building loads.

(*d*), (*e*), and (*f*) The ultimate bearing capacity, q_o, on the surface of a mass of cohesionless soil can be determined by load tests, keeping in mind that it increases approximately in direct proportion to the width of the area. By placing the same loaded area at increasing depths, h, the ultimate bearing capacity, q_h, will increase rapidly as compared with the value of q_o. This increase is due to a greater confinement of the soil underlying the area and to side friction. Thus,

$$q_h = q_o [1 + f(h)] \quad \ldots\ldots (19)$$

in which the function of depth, $f(h)$, determines the increase with depth. The senior author has generally assumed a linear increase with depth, and in 1934 he was instrumental in introducing a modest linear increase into a proposal for a new building code for Boston.[92] More recently he has been led by observations in practice to consider a linear increase too conservative for shallow foundations.

For deep foundations the linear increase always applies, but in the writers' opinion the ultimate bearing capacity becomes so large that other factors than the ultimate bearing capacity control the design. For example, if a load test on an area 2 ft by 2 ft on the surface of a sand shows an ultimate bearing capacity of 4 tons per sq ft, the ultimate bearing capacity at a depth of only 4 ft would be more than 20 tons per sq ft; and, at a depth of 25 ft, more than 100 tons per sq ft, assuming a linear increase according to the conservative Eq. 22 which follows. Therefore, any simplified statement on the increase of ultimate bearing capacity with depth should apply primarily to comparatively shallow depths.

The senior author considers Eq. 3 in its general form the best available expression for the pertinent shallow range. Professor Terzaghi considers it applicable for the range from zero depth to a depth equal to twice the width of a footing. He suggests that the linear formula, Eq. 4, represents the part of Eq. 3 within the range $1 < \frac{h}{R} < 4$. A better agreement with Eq. 3, including validity for the important range from the surface to a depth $h = R$, is afforded by the following equation in which the increase with depth is a parabolic function:

$$q_h = q_o \left[1 + \left(\frac{h}{R} \right)^{1.35} \right] \quad \ldots\ldots (20)$$

In general form this equation may be written

$$q_h = q_o \left[1 + \left(\frac{h}{R} \right)^{m} \right] \quad \ldots\ldots (21)$$

[92] "Proposed New Boston Building Code," by A. Casagrande, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., 1936, Vol. II, p. 311, paragraph (*d*).

The exponent m depends largely on the soil properties and for shallow foundations varies from values larger than 1.0 to almost 2.0.

Most of the discussers expressed their belief in a linear increase of ultimate bearing capacity with depth, which is undoubtedly the simplest rational formulation. The question arises as to what form of linear increase would be suitable for safe application with a minimum of soil investigations. The following linear equation—

$$q_h = q_o \left(1 + \frac{h}{R} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

—has the advantage that it is reliable for very shallow depths and that it can be safely used for any depth, since side friction is entirely neglected. For q_o one may use the results of load tests, or the following well-known formula,

$$q_o = R\,\gamma \tan^4 \left(45 + \frac{\phi}{2} \right) \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which γ is the unit weight of the soil, buoyant below the ground-water surface. The angle of internal friction ϕ of a cohesionless soil can often be estimated from its general character and density.

When greater accuracy is desired for the usual working range from the surface to depths equal to several times the width of a footing, detailed soil investigations will be required. In this case, substitution of the increase along a parabolic curve by a straight line is not justified, and a more accurate formula, such as Professor Terzaghi's Eq. 3, in its general form, should be used.

For the purpose of a simplified statement the writers change formulation (*d*) in the paper to:

> The ultimate bearing capacity of a cohesionless soil increases with the depth of the loaded area beneath adjacent ground surface, approximately according to linear Eq. 22.

Mr. Freeman and the Soil Mechanics Committee of the Los Angeles Section disagree with the writers' statement that in the case of deep foundations on dense sand even the heaviest loading allowed by building codes is exceedingly conservative as far as the strength of the soil is concerned. They refer to the possibility of considerable differential settlement, which is not the issue in this statement. A footing may undergo objectionable settlement from a load which is still entirely safe as far as the strength of the soil is concerned. The occurrence of objectionable differential settlements does not imply that a safe load, as far as the strength of the soil is concerned, was exceeded.

(*g*) and (*h*) In discussing the validity of Hooke's law for cohesionless soils, Mr. Feld is apparently of the opinion that the Boussinesq formula does give satisfactory results for cohesionless soils—at least for small stresses. Mr. Julian states that there is a possibility that a cohesionless mass may have elastic properties when confined under high pressures. The writers agree with Mr. Julian, but believe that for most practical problems the assumption of Hooke's law for cohesionless materials leads to a distribution of soil reactions which are grossly in error.

(*i*) and (*j*) Messrs. Terzaghi and Freeman state that the decrease of settlements with depth may not apply to statement (*j*), depending largely on the effect of construction operations on the soil underlying the area to be loaded.

The writers agree fully with these qualifications. For example, when in an open excavation upward flow of water is allowed to develop, even the most compact and highly consolidated sand may change into a quick condition and become loose and highly compressible.

Within the usual working range of building footings (say, from depths of zero to 10 ft) the decrease of settlements for sand does apply and should be considered when designing such foundations.

In regard to the objections raised in the discussions, the writers propose the following modification of (*j*):

> With increasing depth of a loaded area, the settlement due to a given unit load will decrease appreciably only if the soil under the loaded area remains undisturbed. When disturbance is inevitable, the settlement may be assumed to be independent of depth.

The Liberty Mutual and New England Mutual Life Insurance Buildings

Building I, a foundation plan of which is shown in Fig. 17, is a ten-story tower-shaped office building having two setbacks. The plan area of the first two floors and basement is approximately 32,000 sq ft; the area of the third through sixth floors, approximately 20,000 sq ft; and the area of the seventh through tenth floors, approximately 10,000 sq ft. The dead load of this building, as computed from data furnished by the Turner Construction Company and based on actual quantities used in construction, is 39,500 tons. This dead load was approximately 5% less than the design dead load. The actual live load, as determined from an inspection of the building during which representative items of live load were weighed and unit intensities computed, is 1,500 tons. The effective weight of the soil removed to form a general basement, and in addition a sub-basement under the more heavily loaded ten-story section, is 35,000 tons. The net increase in load for the entire area is 6,000 tons.

Building II, a foundation plan of which is shown in Fig. 18(*a*), covers a plan area of approximately 66,000 sq ft. The section to the left of line EF and that to the right of GH are four-story sections. The area bounded by EGJI is occupied by a ten-story section; areas IMNK and OJLP, by two-story sections; and area FKLH, by a one-story section, with the exception of the center part, approximately 60 ft in length, which carries two stories. In the center of the building is an ornamental tower section approximately 40 ft square which rises 125 ft above the ten-story height. The dead load of this building, as determined from data furnished by the Turner Construction Company, and again based on actual quantities used in construction, is 110,000 tons. In this case 104,000 tons was the design dead load. The estimated actual live load is 6,000 tons. The effective weight of soil removed from this area is 127,000 tons. The estimated weight of two buildings which formerly occupied this site was 16,700 tons. The total weight removed from this area exceeds the weight applied in the form of the completed structure. This additional

weight was removed to provide for dead-load increases and live-load increases in additions contemplated for the future and for which the foundations were designed.

The widths of all wall footings for building II were designed to provide for additional loads contemplated for the future. In many respects, however, the existing load distribution is more critical, and it is anticipated that, when the more lightly loaded sections are increased in story height, the differential settlements will diminish.

Both foundations were designed for dead loads plus actual live loads which were estimated as accurately as possible.

The thickness of the crust of hard and stiff clay was thoroughly explored during the construction of both buildings. For building I, the thickness was ascertained by making one hand-auger boring in the bottom of every caisson. The diameters of the caisson bells were proportioned in relation to the actual thickness of the crust. For building II numerous auger borings were made during the construction period to determine the variations in thickness of the crust. As a result of these explorations, it was decided during construction to raise the elevation of the base of the Berkeley Street exterior wall so as not to cut into the hard crust and unduly reduce its thickness. At this location the elevation of the base was slightly higher than at other places in the building area.

The design of the outside foundation walls for building II for horizontal forces was largely determined by the hydrostatic pressure. To this was added earth pressure at rest, buoyant below ground-water level. The temporary sheeting and bracing was designed on the basis of triaxial tests on undisturbed samples of the organic silt, making due allowance for a modest yield. Later the earth pressure was measured directly and found to check well with the design values.

Mr. Woolf takes exception to the specification for removing hard clay from under the sub-basement slabs to prevent them from bearing directly on hard material. This was necessary to assure that all loads would be transferred through the wall footings. Otherwise it would not have been possible to determine the soil reactions for which the slabs should be designed. Even if these pressures had been known, it would have been uneconomical to permit the slabs to carry the building load. If the crust of hard clay were not available, then a continuous mat would have been necessary, as emphasized by Mr. Freeman.

The main reasons for designing eccentric footings for the outside walls were: (1) Simplification of construction, and (2) more uniform distribution of the contact pressures on the base, considering water and earth pressure against the walls. The Building Department did not permit transmitting the earth pressure horizontally through the floor systems, thus necessitating transmitting these forces through the wall footings.

From the observed deflections, because he desired a comparison with the actual design, Mr. Woolf spot-checked the design of an interior wall for building II and computed the amount of reinforcing steel required to be 60 sq in. These walls were provided with 100 sq in. of steel in both the top and bottom.

Several discussers expressed a preference for the design of the foundation of building II over that of building I. When comparing these designs, the difference in the design of the superstructure should be kept in mind. The type of foundation designed for building II was made possible by the "columnless type" of building design.

Settlements.—The most recent settlement observations of buildings I and II were conducted after 6.5 and 3.8 years had elapsed, respectively, since installation of the observation points. The progress of the settlements is consistent with those reported in the paper. Table 2 summarizes the essential values.

TABLE 2.—SUPPLEMENTARY REPORT OF SETTLEMENT

Observation	Years elapsed	
(a) BUILDING I, LIBERTY MUTUAL INSURANCE COMPANY BUILDING		
Elapsed period, in years	4.3	6.5
Maximum settlement for point A93, Fig. 17 (curve I, Fig. 15(*b*))	1.85	2.04
Maximum differential settlement (curve I, Fig. 15(*c*))	1.02	1.18
(b) BUILDING II, NEW ENGLAND MUTUAL LIFE INSURANCE COMPANY BUILDING		
Elapsed period, in years	2.1	3.8
Maximum settlement (curve II, Fig. 15(*b*) and Fig. 23)	1.80	1.98
Settlement of point 9, Fig. 22	1.06	1.17
Maximum differential settlement (curve II, Fig. 15(*c*))	1.18	1.29

Referring to Fig. 20, Mr. Freeman concludes from the available observations that point 5, building II, might have risen considerably more than 2.9 in. before the rise was arrested by the weight of the concrete. The additional rise of point 5 which occurred in the period from October 12 to October 23, 1939, is estimated by the writers from a study of movements of points 4 and 6, Fig. 20, to be between 0.3 and 0.4 in. In this connection it should be emphasized that point 9 rose slightly after the foundation slabs and walls were installed at this location, due to an increase in hydrostatic pressure on the slab.

Mr. McAlpine inquires whether any of the adjacent buildings were adversely affected during or after construction. Great care was exercised in frequently observing elevations of observation points established on all structures in close proximity to the site of operations. No movements could be detected in any building surrounding the area. In this respect the soil conditions in Boston are more favorable than in some other cities where such deep excavations would invariably result in lateral flow of the soil and settlements of adjacent buildings.

The lack of symmetry in the settlement contours of the central area of building I, Fig. 17, is due to a nonuniform depth of excavation. The area enclosed by the heavy black line defines the sub-basement area. A depth of soil varying from the basement level to the sub-basement level was excavated from the area enclosed by the dot-dashed line on the Berkeley Street side of

the sub-basement section. Had it been possible to excavate more soil at location A, the settlement pattern would probably have been symmetrical.

In reply to the requests by Messrs. Krynine and Burmister, for a comparison of computed and actual settlements, the following is quoted from a comprehensive investigation by the junior author:[93]

> "The analysis of data for six buildings located within the confines of Boston has suggested the following conclusions:
> "(1) Settlements due to consolidation, as computed from information disclosed by boring records, numerical data obtained from carefully performed consolidation tests of representative undisturbed samples, and stresses computed on the basis of the elastic theory, varied between values equal to and up to twice the magnitude of the observed settlements.
> "(2) Conservative estimates of the predicted differential settlements are obtained if, in computing the stresses by the elastic theory, the following simplifying assumptions are made: (*a*) The soil above the plane of loading and adjacent to the loaded area does not influence the distribution of the load below the plane of loading. (*b*) A structure is relatively flexible and does not redistribute the load as settlement occurs. It was generally observed that the ratio of the observed settlement to the computed settlement was low for points of large settlement and high for points of small settlement."

Recent investigations indicate that the excess of predicted settlements over actual settlements can be further decreased by considering more carefully the effect of preconsolidation of a substantial thickness of the clay stratum by drying.

Control of Settlements by Combining Flotation and Stiffness

The interesting historical remarks by Professor Terzaghi on early applications of the principle of flotation to the design of foundations, and his discussion of the time lag between the inception of a new idea and its general acceptance, explain why the profession has so overlooked this method for almost one hundred years that it had to be rediscovered. However, he still leaves unanswered the question: Why is not more use made of this method in the design of modern buildings? The writers believe that the principle of flotation by itself is not sufficient to design satisfactory floating foundations for large buildings on soft ground. Just as ships require stiffness to withstand fluctuating support and variable loading, so must buildings floating on soft ground be designed to withstand nonuniform support and loading. In this connection, conclusion (5) of the paper deserves to be underscored. The problem of the design of such foundations is by no means solved in a completely satisfactory manner. Cooperation between architect and engineer should lead to novel and more practical designs in which full use is made of the superstructure in obtaining adequate stiffness.

If those structures are excepted in which functional requirements demand excavation at least equal to the weight of the structure, the method of flotation would probably be worth while only for very heavy structures, or structures with nonuniform load distribution.

[93] "Observations and Analysis of Building Settlements in Boston," by R. E. Fadum, a thesis submitted to the Faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering, pp. 237 and 238.

Professor Zeevaert describes the application of an interesting and novel idea for compensating for the dishing effect by decreasing the depth of the excavation toward the edges of the building, thus assuring a more uniform stress distribution in the underlying soil. This method is theoretically sound and it would be worth while to have the details of this design made available to the profession. Professor Zeevaert expresses the opinion that heaving due to excavating in very soft clay is mainly caused by lateral plastic flow rather than by vertical expansion of the underlying clay. This is probably correct for Mexico City, although at the site of building II it was proved that the ground lost because of minor subsidence of the immediately adjacent street surface was of the same order of magnitude as the volume lost because of outward movement of the sheeting. The volume of the heave over the entire building area was of a much larger magnitude.

The writers concur with Professor Tschebotarioff's statement that, under different loading and soil conditions than those at building II, recompression following swelling may not be completed during the construction period.

Mr. Wilson suggests an interesting foundation design, which is intended more nearly to satisfy the requirement that the building load should be removed and applied simultaneously.

Floating Foundations Versus Deep Foundations

The writers agree fully with Mr. White that deep foundations, which transfer load through soft clay on to hard underlying material, are preferable to floating foundations, when the cost of the former type is only a little higher than that of the latter. However, it was not the purpose of the paper to discuss foundations in which settlements are eliminated by carrying the loads on practically incompressible materials, but to present some applications of soil mechanics to the design of foundations which transmit load on to soft soil. The question as to when deep foundations should be selected was also beyond the scope of the paper.

For both buildings described, a pile or caisson foundation to rock would have been much more expensive than the floating foundation. For building II comparative cost studies were made for steel piles to rock, for caissons to rock, and for the floating foundation. These studies left no doubt that a deep foundation would be much more expensive than the floating foundation. A cost analysis of building II, after its completion, showed a unit cost for the substructure of $5.50 per ton load. This value may be compared with the actual cost of considerably more than twice this amount for a foundation to rock, consisting of open-end steel pipe piles, which was installed at a site in Greater Boston where similar subsoil conditions prevailed. When considering the value of the additional basement space provided by the floating foundation, the comparison is still more in favor of the floating foundation.

It was intimated in the discussions that only on the basis of detailed soil investigations, as conducted for these two buildings, would it be possible to design satisfactory floating foundations. However, most of these investigations served to analyze the settlements for foundations of a conventional type.

These studies showed that such foundations would result in excessive differential settlements. Could it have been possible to decide beforehand on a floating type of foundation, most of these soil investigations would not have been necessary.

Concluding Remarks

In retrospect, several conflicting statements appear rather prominently in the discussions. On close scrutiny one finds that many of these conflicts are caused by a scarcity of reliable observations.

The designer is often confronted with the dilemma of whether to put more trust in observations whose accuracy he may have reason to doubt or in a theory which at best is a crude approximation. A theory can be modified to yield such a sufficiently wide range of results as may reasonably be expected to include the actual conditions. An observation is an empirical fact with which one cannot argue, unless one is thoroughly familiar with all the details.

Many observations from field and laboratory are subject to errors which render them valueless or actually misleading. For example, only a small percentage of measurements conducted with pressure cells can be considered reliable. The measurement of pore-water pressure in clays is assuming increasing importance, but it remains to be proved whether any of the existing measuring devices really give accurate results. Even the simplest field measurement (namely, the observation of settlements) has occasionally been subject to serious errors. Undisturbed samples of the same clay have been sent to different laboratories for shear strength tests. The results were so far apart that an objective observer must greatly wonder about the confidence with which such data were used in "accurate" stability computations. Therefore, let the frequently heard call for more observations and research on the behavior of earth works and foundations be tempered by emphasis on quality rather than on quantity.

Acknowledgment.—The data from explorations of the clay deposits at the site of the Mystic Power Station are used with the permission of the Boston Edison Company and Jackson and Moreland, Engineers, for whom the writers made these investigations. The writers wish to thank R. P. Burden, of the Staff of the Harvard Graduate School of Engineering, for his assistance in the preparation of material and review of the text for this closure; and J. A. Delaney, of the staff of the Harvard Soil Mechanics Laboratory, for the performance of tests.

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DISCUSSIONS

EFFECT OF TURBULENCE ON SEDIMENTATION

Discussion

BY H. B. SOUTHWOOD, L. STANDISH HALL, AND WILLIAM E. DOBBINS

H. B. SOUTHWOOD,[13] ESQ.[13a]—Although limited in scope by the necessity for simplification in order to arrive at equations possible of solution, the author's study provides additional verification and extension of principles forming the basis for the largely qualitative analyses on which the engineer must often depend when he deals with problems of open-channel flow involving sediment transportation and deposition. The definition of any of the factors relating to these phenomena, however circumscribed may be the conditions under which the analysis is made, is a step forward and eliminates at least some part of the guesswork included in the process generally termed "engineering judgment" or "engineering intuition." The author's rational approach, resulting in an expression of the relations between such factors as concentration at any point, pickup, hydraulic conditions at the bed, and characteristics of the bed material, certainly constitutes such a forward step. Direct application of the results to the solution of practical problems must necessarily await the formulation of methods whereby turbulence can be classified, evaluated, and measured. Further investigations of turbulent suspensions may result in the establishment of relations that will permit the measurement of turbulence by means of sediment observations. The development of a relatively simple field technique based on such relations would be an invaluable tool in the study of natural streams and other open channels.

Regarding the observed lack of conformity between experimental data and theoretical values as plotted in Fig. 7, the two factors mentioned by the author (upward transportation of material when the cylinder was dropped, and pickup of the material lodging on the grid surface) are considered to have had the described effects. However, at least two additional factors are believed to have contributed to the existence of observed concentrations in excess of plotted theoretical values.

NOTE.—This paper by William E. Dobbins was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by A. M. Gaudin; May, 1943, by John S. McNown; June, 1943, by Messrs. H. A. Einstein, and Thomas R. Camp; October, 1943, by Messrs. Garbis H. Keulegan, C. W. Thornthwaite, and A. A. Kalinske; January, 1944, by E. R. Van Driest; and February, 1944, by R. Schuhmann, Jr.

[13] Chf., Hydr. Section, Dept. of Public Works, State of Louisiana, Baton Rouge, La.

[13a] Received by the Secretary January 31, 1944.

(1) The use of an arithmetical average value for settling velocity in the computations may result in erroneous positions of the theoretical curves. A range in values of from 0.150 cm per sec to 0.344 cm per sec appears rather wide. Analyses of individual samples obtained during the course of the run probably would have given some insight into the effect of this factor.

(2) The method of sampling is subject to errors which may be sufficiently great to account for most of the deviation noted in Fig. 7. The description of sampling procedure indicates that flow through the siphon was in progress for 12 sec to 18 sec prior to beginning of collection of each sample. This should be a sufficient period for the establishment of a field or zone of undetermined lateral and vertical extent in which the pattern of turbulence would be distinctly altered by the velocities of the converging flow entering the siphon tube. Since these sustained velocities undoubtedly had substantial upward components throughout the major portion of the affected field, it follows that concentrations of suspended material entering the sampling tube would be in excess of those existing at the same elevation in other portions of the container.

L. STANDISH HALL,[14] M. AM. SOC. C. E.[14a]—Although the tests and theory described in this paper are primarily intended for application in the design of settling basins, the author indicates that the same principles may be used for studies of sediment load in natural streams. A modification of the formulas would be required before such a general application could be made. The author states that "the simplifying assumptions involved in Eqs. 17 may introduce large errors when applied to the case of a natural stream." This is undoubtedly true.

The settling rate of suspended particles conforms with Stokes' law (25).[14b] The lucite used by Ensign Dobbins in the experiments had a specific gravity of 1.194 as compared with a specific gravity of 2.65 for normal stream sediment. The diameter of the lucite particles ranges from 0.138 mm to 0.216 mm. By Stokes' law, these diameters would correspond to diameters of 0.048 mm to 0.075 mm for stream sediment of the usual specific gravity.

Suspended sediment in stream channels normally is composed predominantly of silty loam with some percentage of clay and some percentage of fine sand. Medium or coarse sand, gravel, and boulders ordinarily move on the stream bottom as bed load. The analysis of suspended-stream sediment indicates that normally 70% to 80% is of a smaller diameter than the equivalent of the lucite; also, that from 10% to 20% is of larger diameter. Therefore, the tests are indicative of the performance of larger sizes of suspended sediment in streams.

In the transportation of sediment in stream channels, the movement of the bed load absorbs a part of the momentum. This action undoubtedly influences the relation between the suspended load and the total load, so that the latter depends on the relative quantity of each type of load carried in the stream

[14] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

[14a] Received by the Secretary February 2, 1944.

[14b] Numerals in parentheses, thus: (25), refer to corresponding items in the Bibliography (see Appendix 1) of the paper, and at the end of discussion in this issue.

channel. Without some such explanation, it is difficult to interpret the results of actual measurements of silt load. For example, the maximum concentration of sediment usually occurs prior to the peak discharge; and the sediment load carried by a stream varies between a rising and a falling stage, being greater in the former, and also varies with the same discharge between different storms (26).

More study is required on the larger problem of the distribution of sediment particles in a natural stream channel because of the greater number of variables that must be introduced. Notwithstanding the limitations of the formulas, Ensign Dobbins' paper is a valuable contribution to the theory of sedimentation.

WILLIAM E. DOBBINS,[15] JUN. AM. SOC. C. E.[15a]—Many interesting and important suggestions have been revealed in the discussions of this paper. The ultimate solution to this difficult problem is still to be found. The principal difficulties lie in obtaining the solution to the general form of the differential equation and in finding the complete relationship between the rate of pickup from the bottom and the factors that control it. Professor Kalinske has reported progress on the latter problem.

If these difficulties can be overcome, the resulting equations undoubtedly will be very cumbersome. However, it is much better to introduce simplifications after the correct solution has been found than before. Long and tedious calculations need be made only once since the results can be preserved in the form of tables or charts. The writer hopes that he may have contributed some ideas that will help others who are working on this and other related problems.

Bibliography.—

(25) **"Applied Fluid Mechanics,"** by Morrough P. O'Brien and George H. Hickox, McGraw-Hill Book Co., Inc., New York, 1937, p. 189.

(26) **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** *Report No. 1* entitled "Field Practice and Equipment Used in Sampling Suspended Sediment," St. Paul Engr. Dist. Sub-Office, Hydr. Laboratory, Univ. of Iowa, Iowa City, August, 1940, pp. 85 and 90.

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[15] Ensign, CEC-V(S), U.S.N.R., Woburn, Mass.

[15a] Received by the Secretary February 2, 1944.

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DISCUSSIONS

FLOW AROUND BENDS IN STABLE CHANNELS

Discussion

BY C. A. MOCKMORE, M. AM. SOC. C. E.

C. A. MOCKMORE,[42] M. AM. SOC. C. E.[42a]—Very properly, Professor Robertson records that well-known textbooks on hydraulics present data and make statements inferring that the filament of maximum velocity is always at the outside of the bend and that, in fact, "the filament of maximum velocity is initially, and, for most of the way around the bend, near the inside of the bend." The maximum velocity filament is at the inside of the bend initially, but is not there near the end of the bend, either in an open or a closed channel. In addition, Professor Robertson presents the results of some very fine experiments which he conducted at the University of Illinois in Urbana with closed pipe bends. Fig. 18 is especially interesting and valuable.

An attempt was made by the writer to evaluate α and β as presented by Messrs. Johnson and Selim. The reason for leaving this study out of the paper is given in their comment—"the data for only three stages are not enough to warrant any conclusions."

Mr. Nelidov applies the theory of gyroscopic motion to flow of a fluid around a bend. The theory works well with solid bodies, but it is difficult to form a clear picture of its application to a stream of water flowing around a bend where there are myriads of exceedingly small particles, molecular in size and not rigidly affixed to one another.

Several interesting questions on the "strength" of the spiral in an open channel are formulated by Messrs. Woodward, Howe, and Posey. It is a delight to contemplate the questions which these discussers would have asked had they been present when the apparatus shown in Fig. 1 was in operation. All the answers must await further researches.

There seems to be a slight misconception of the assumptions for the mathematical analyses. The writer does not assume "that the spiral flow comes suddenly into existence at the point of tangency and remains constant around

NOTE.—This paper by C. A. Mockmore was published in March, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. J. M. Robertson, and Joe W. Johnson and M. A. Selim; October, 1943, by I. Nelidov; and December, 1943, by S. M. Woodward, J. W. Howe, and C. J. Posey.

[42] Prof. and Head, Dept. of Civ. Eng., Oregon State College, Corvallis, Ore.

[42a] Received by the Secretary February 2, 1944.

the bend," but that a particular flow condition exists at a cross section of the bend of the stream, such as that in Fig. 7. Then the curves in Fig. 9 give a graphical representation of the paths various particles of water would take if they were permitted to continue that behavior in a tangent to the bend of the channel. At any other downstream section an entirely new set of flow conditions may obtain.

The writer has read the article by Messrs. Blue, Herbert, and Lancefield,[36] describing a detailed study of a full-sized river bend, and appreciates not only the value of the work but also the hard labor and the occasional hazards incident to collecting such data, since on three different occasions he was spilled out of a boat into a mountain stream (once at flood stage) while attempting to collect river flow data. The writer firmly believes that the fundamentals of stream flow can best be studied in the laboratory, and second best in the field.

[36] "Flow Around a River Bend Investigated," by F. L. Blue, Jr., J. K. Herbert, and R. L. Lancefield, *Civil Engineering*, May, 1934, p. 258.

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DISCUSSIONS

THE QUEENS MIDTOWN TUNNEL

Discussion

BY OLE SINGSTAD, M. AM. SOC. C. E.

OLE SINGSTAD,[26] M. AM. SOC. C. E.[26a]—The discussions, and the questions raised in them, have contributed much to the value of the paper. The writer appreciates the complimentary remarks regarding the paper made by the discussers, and he is particularly gratified that no less than six of his former associates in the planning and building of the Holland Tunnel have taken part in the discussion.

Mr. Fluhr relates the fundamental differences between the geology of the East River and that of the Hudson River, describing the causes of the variations in character of rock and the porous nature of the overburden in the East River which account for the increased hazard, slower progress, and higher cost of building a tunnel under the East River as compared to the Hudson River.

Mr. King suggests an interesting experiment to determine whether a soft plastic backfill such as clay, with perhaps a small admixture of bentonite outside of the tunnel lining in rock, instead of grout and gravel, would make the lining more nearly watertight. If practicable, this would undoubtedly be an aid to the watertightness of the tunnel. This kind of fill, however, would greatly reduce the passive lateral support of the surrounding material. The light cast-iron lining is designed to withstand, in addition to the hydrostatic pressure, a load of 10 ft of loose rock above the top of the tunnel, assuming full support at the haunches by grout and gravel. If this full support is removed and a plastic material substituted under this loading condition, excessive tensile stresses and deflections are indicated. To meet such a contingency, permanent blocking or supports might be constructed between the outside of the lining and the rock as required, although this would not always be practicable.

Mr. Wagner makes an analysis of the operating costs of the Queens compressor plant of the Queens Midtown Tunnel. The observation is made that it

NOTE.—This paper by Ole Singstad was published in March, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1943, by Messrs. Thomas W. Fluhr and Howard L. King; June, 1943, by Messrs. Francis V. Wagner, William J. Wilgus, and S. A. Thoresen; September, 1943, by Messrs. H. G. Cruthers, Leo Geenens, Charles E. Conover, Edward Levy, and Robert F. Legget; November, 1943, by Orrin L. Brodie; and February, 1944, by A. A. Eremin, and Arthur S. Tuttle.

[26] Chf. Engr., New York City Tunnel Authority, New York, N. Y.

[26a] Received by the Secretary February 2, 1944.

is the fixed charges that are important in considering construction power-house costs and that savings made in them are of more importance than economies in operating expenses.

The tunnel engineer's chief responsibilities with regard to the power house are (1) to provide a compressor plant of sufficient low-pressure air capacity to be adequate for the worst conditions anticipated, and (2) to assure continuous operation. Economy of fixed costs and operating costs must be considered, but it is of secondary importance to these requirements. An experienced tunnel engineer may be able to make a reasonable estimate of the required average compressed air capacity of a proposed plant, but such a capacity would generally not be safe to install.

The low-pressure air capacities specified for the Queens Midtown Tunnel were determined primarily on the basis of the experiences during construction of the Pennsylvania Railroad Tunnels, which had been constructed near-by through somewhat the same materials as the Queens Midtown Tunnel. Even with this basis of comparison it was difficult to anticipate the Queens Midtown Tunnel requirements, as its diameter (31 ft) differed considerably from that of the Pennsylvania Railroad Tunnels (23 ft). To insure continuous operation, three independent power sources were provided for each compressor plant of the Queens Midtown Tunnel.

Actual experience in driving the Queens Midtown Tunnel demonstrated that the low-pressure air capacity provided on the Manhattan side was none too great, whereas the low-pressure air capacity of the plant on the Queens side might have been made less without jeopardizing the work. The writer cannot agree with Mr. Wagner's statement, however, that the "Queens side of this tunnel was in exceptionally good ground for tunneling," although the ground was much better than that encountered on the Manhattan side and generally somewhat better than had been anticipated. Most of the large-diameter, shield-driven tunnels constructed prior to the Queens Midtown Tunnel had been driven in what could correctly be described as exceptionally good ground for tunneling, consisting largely of impervious silt and clay, vastly superior for tunneling to the ground encountered on the Queens side of the Queens Midtown Tunnel. Among these tunnels the writer would include the Holland, Lincoln, East Boston, and Antwerp tunnels and the shield-driven sections of the Detroit-Windsor vehicular tunnel.

Predetermination of the required capacity of a compressor plant to meet exactly the low-pressure compressed air requirements is practically impossible, especially for a tunnel diameter as large as that of the Queens Midtown Tunnel and with similar ground conditions. The tunnel engineer's judgment, in determining the proper capacity to be installed, must be guided by considerations of a margin of safety to meet emergencies which are more important than the fixed and operating costs.

Colonel Wilgus raises several pertinent questions. The quantity of leakage during winter months is too small to be measured, and during the summer there is no evidence of any leakage. The objection to such a small amount of leakage in the tile-finished interior of a modern vehicular tunnel is largely, if not entirely, on the basis of appearance.

The estimated cost of the Queens Midtown Tunnel compared with the actual cost is the estimate made at the outset by the writer when acting as consulting engineer to the Tunnel Authority before the engineering staff was organized.

TABLE 6.—Comparison of Costs and Construction Indexes

Item No.	Description	Holland Tunnel	Lincoln Tunnel		Queens Midtown Tunnel
			South tube	North tube	
	Contracts let	1922	1934	1937	1937
	(*a*) Cast-Iron Lined Shield-Driven Tunnel				
1	Dollars per linear foot	1,430	1,235	1,657	3,159[a]
2	Dollars per cubic foot	1.83	1.63	2.20	4.18
3	Percentage cost per cubic foot	100	89	120	228
	(*b*) Basic Labor Rates				
4	Muckers, dollars per day	7.00	10.00	11.00	11.00
5	Ironmen, dollars per day	8.00	10.00	11.00	11.00
6	Miners, dollars per day	8.50	11.00	12.00	12.00
7	Average, all sandhogs	8.00	10.50	11.50	11.50
8	Index number (Holland Tunnel, 100)	100	131	144	144
9	Tunnel laborer, dollars per hour	0.60	0.75	0.825	0.825
10	Form setter, dollars per hour	1.125	1.40	1.40	1.40
11	Average, cleaning and concreting (Items 9 and 10)	0.862	1.07	1.11	1.11
12	Index number (Holland Tunnel, 100)	100	125	129	129
	(*c*) Basic Materials				
13	Cement, in dollars per barrel[b]	2.43	2.48	2.07	2.07
14	Steel shapes, in dollars per hundred weight	1.70	1.78	2.21	2.21
15	Cast-iron lining, in dollars per ton delivered	35.00	56.80	61.20	60.70
	(*d*) Index Numbers, Engineering News-Record				
16	Construction cost (1913 = 100)	174.45	198.10	234.71	234.71
17	Construction cost (1922 = 100)	100	113	140	140
18	Materials cost (part of Item 16)	79	88	103	103
19	Materials cost (1922 = 100)	100	111.5	130.6	130.6
	(*e*) Average Labor Wage, New York City				
	Skilled Labor:				
20	Dollars per hour	1.17	1.52	1.71	1 71
21	Index number (1922 = 100)	100	130	146	146
	Common Labor:				
22	Dollars per hour	0.60	0.8475	0.76	0.76
23	Index number (1922 = 100)	100	141	127	127
	(*f*) Tunnel Cost Indexes (Holland Tunnel = 100)				
24	Labor only	100	130	141	141
25	Material only	100	156	166	164
26	Labor and material	100	143	153.5	152.5

[a] The contractor's payrolls aggregated $1,287 per foot of cast-iron lined shield-driven tube, exclusive of shafts and including concrete lining placed under river contract on the Queens Midtown Tunnel, or as much per linear foot as the total cost of the same work on the south tube of the Lincoln Tunnel. [b] Delivered in New York City.

It was based on preliminary plans and made at the time of application to the PWA for funds for the project.

The cost per linear foot of cast-iron lined shield-driven tunnel, exclusive of shafts and including concrete lining as placed on the river contracts, is given in Item 1, Table 6(*a*), for the Holland Tunnel, as of 1922, the south tube of the Lincoln Tunnel, as of 1934, the north tube of the Lincoln Tunnel, as of 1937, and the Queens Midtown Tunnel, as of 1937. The respective costs per cubic foot of tunnel are given in Item 2 and, on the basis of 100% for the Holland Tunnel, the respective percentage costs are given in Item 3, Table 6(*a*).

The index numbers in Table 6(*f*) are based, for material (Item 25), on the cost of cast-iron lining (Item 15, Table 6(*c*)) and concrete per cubic foot of tunnel and, for labor (Item 24), on the basic labor rates for sandhogs (Item 7, Table 6(*b*)) and for the labor used in cleaning the tunnel and concreting (Item 11). Using the index numbers in Item 26, Table 6(*f*), the costs of the Queens Midtown Tunnel, reduced to Holland Tunnel conditions (1922), were as follows:

	Dollars per cubic foot (Holland Tunnel basis)	Percentage comparison (1922 = 100)
Holland Tunnel	1.83	100
Lincoln Tunnel, south tube	1.135	62
Lincoln Tunnel, north tube	1.43	78
Queens Midtown Tunnel	2.74	150

In other words, if contracted for under labor and material rates prevailing at the time of the construction of the Holland Tunnel, the river section of the south tube of the Lincoln Tunnel would have cost 62% as much as the Holland Tunnel per unit, that of the north tube of the Lincoln Tunnel 78%, and that of the Queens Midtown Tunnel 150%.

It should be noted that these differences in cost, as adjusted for the labor and materials indexes, are applied to the shield-driven river sections only and can be accounted for practically in their entirety on the following basis:

The Holland and Lincoln tubes were constructed through substantially the same kind of material, namely, Hudson River silt. As the Holland Tunnel was the first large tunnel built under the Hudson River, there was much uncertainty as to the behavior and stability of such a large tunnel in this soft material during the construction period. Therefore, the bid prices were high, but in view of the uncertainties, not unduly high. During the driving of the Holland Tunnel, after considerable experimentation, it was found that only about 20% of the material displaced by the tunnel had to be taken in through the face of the shield, and approximately 80% could be pushed aside by advancing the shield through the silt. This had two distinct advantages—namely, fast progress with correspondingly reduced cost, and stability. It was found that by leaving the 20% of the displaced material in the bottom of the tunnel until shield driving was completed, the weight increased the stability of the tube which, during the disturbed condition attending the shield driving, had a buoyant tendency. When the south tube of the Lincoln Tunnel was being built, this exploratory or pioneering work on the Holland Tunnel was used to

great advantage by the contractor in adapting his tunneling methods from the start to the experience gained on the former project. The contractor's manager-engineer on the Lincoln Tunnel was a former resident engineer in charge of building the Holland tube on the staff of the chief engineer of the Tunnel Commissions. In the building of the Lincoln Tunnel the contractor developed a mechanical bolt-tightening device which greatly increased progress, since no muck was transported out of the tunnel until after the tube construction was completed. The progress was then a function of the speed with which the iron rings could be erected and bolted together. If another similar tunnel in material such as Hudson River silt were to be built by experienced tunnel engineers and contractors, the cost of the under-river section of the Lincoln Tunnel south tube would be a truer basis for cost estimating than that of the Holland Tunnel.

In the case of the Queens Midtown Tunnel, which is the last one to be built, the cost as adjusted for the labor and material index (Item 26, Table 6(*f*)) is very much in excess of that of either of the Hudson River tunnels, and this can be ascribed in its entirety to the extremely difficult and hazardous ground conditions existing on a great part of the Queens Midtown Tunnel site, which impeded progress and caused higher air pressures, with correspondingly reduced daily working hours for labor with increased daily rates.

The contract for the north tube of the Lincoln Tunnel was let under approximately the same labor and material cost conditions as the Queens Midtown Tunnel shield-driven section. The fact that this tube was driven alongside the already completed south tube accounts for its higher adjusted cost as compared with the south tube.

Mr. Thoresen states that careful thought has been given to providing the traveling public with an "attractively finished, well lighted, pleasant, and comfortably ventilated subaqueous highway." The writer has long been an advocate of this kind of treatment of vehicular tunnel projects, as reflected in the development of the use of a tile finish on the walls of the Holland Tunnel. At the time of the building of that tunnel, no tile was manufactured in the United States which was deemed suitable for the severe moisture and temperature conditions that would prevail in a tunnel in this climate, and an extensive research program was conducted with the assistance of the late George H. Brown, professor and head of the Department of Ceramics at Rutgers University, New Brunswick, N. J. Extensive laboratory and field tests were made on a large number of tile samples manufactured in the United States and abroad. This resulted in specifications which produced a wall tile not only pleasing in appearance but easy to clean and durable. One half of the tile used on the walls of the Holland Tunnel was imported and the other half was manufactured in the United States. The research work on tile and the requirements specified for the Holland Tunnel have stimulated the domestic tile industry so that this high quality tile is now considered a standard product.

The trench and pre-cast type of tunnel design and construction, on a scale larger than any heretofore undertaken, is contemplated for the Narrows Tunnel between Brooklyn and Staten Island, where conditions indicate that this method is both practicable and economical.

Both Mr. Cruthers and Mr. Geenens correctly state some of the reasons for the greater economy in the operation of the Queens Midtown Tunnel as compared to the Holland Tunnel.

Mr. Conover very appropriately emphasizes the point that the Queens Midtown Tunnel is merely one link in an east-west express highway route running through mid-Manhattan and extending far out on Long Island and into the State of New Jersey. From its very inception, in 1926, all those who had made a comprehensive study of traffic conditions in the metropolitan area realized the necessity for such a through highway. This route comes under many jurisdictions—the state highway departments of the two states, the parkway authorities, the borough presidents of the various boroughs, the Department of Public Works of the City of New York, and the two authorities charged with the responsibility of financing and building the river crossings.

Although, as of 1944, the two principal links in this route—the Lincoln Tunnel under the Hudson River and the Queens Midtown Tunnel under the East River—have been in operation for a period of six and three years, respectively, very little else has been done to complete this through route. In fact, only about two miles of modern highway have been built—namely, that part immediately east of the Queens plaza of the Queens Midtown Tunnel which was completed by the City of New York simultaneously with the opening of that tunnel.

Mr. Conover expresses the hope that the New York City authorities will include a crosstown tunnel connecting the two river tunnels as a postwar project. The writer is in accord with this hope. It would be a rational development of the city's highway system. Other missing links in this through route, particularly extensions farther out on Long Island, should be included.

During the past decade, 1933–1943, New York City has been preoccupied with the building of parkways and high-speed roads for the exclusive use of "pleasure" traffic; that is, passenger cars. Commercial vehicles of every kind have been excluded. Scores of millions of dollars have been spent for this type of highway, with the result that there has not been enough money left to provide modern highway facilities for business traffic. Commercial traffic is still shackled with the stop-and-go lights almost as much as it was twenty years ago. It may have been assumed by those responsible for this policy that, by removing a large amount of passenger cars from the streets and placing them on express highways, there would be that much more room for commercial traffic on the streets with the stop-and-go lights. Experience has demonstrated, however, that this is only partly true because, with the new facilities for the fast movement of passenger cars, there is also a corresponding increase in "pleasure" driving. It is to be hoped that much more attention will be given in New York City to high-speed roads for mixed traffic than has heretofore been the case; otherwise the increased cost of doing business here due to traffic delays is certain to have adverse effects.

Mr. Conover refers to the West Side Plaza and the East Side Plaza developed for the Midtown Manhattan Tunnel by cooperation, in the first case by the engineers of the Board of Transportation and those of the Port of New York Authority, and in the second case by the engineers of the Board of Transporta-

tion and those of the Borough President of Manhattan. He states that the East Side Plaza adopted by the New York City Tunnel Authority differs radically from the plan developed by the Board of Transportation. This is also true of the West Side Plaza. The plan developed jointly by the engineers of the Port Authority and the Board of Transportation and recommended for adoption in April, 1931, by the chief engineer of the Port Authority located the four exits and entrances of the twin Hudson River tubes and of the Manhattan tubes on the easterly line of 10th Avenue in the two narrow blocks between 37th and 39th streets. The writer, who at that time (1931) was one of several consulting engineers to the Port Authority, after analyzing that plan, pointed out that such an arrangement would create unsatisfactory traffic conditions, with resultant congestion and confusion on the four tunnel approaches as well as on the adjacent city streets, particularly on 10th Avenue. The plan violated what the writer considers a fundamental principle—namely, that eastbound traffic and westbound traffic should be separated as much as possible. The writer also pointed out that the plan was unnecessarily costly. As a result of that analysis the plan was not adopted and a re-study was made by the staff of the chief engineer of the Port Authority. In September, 1931, that staff produced a new plan eliminating these objectionable features as well as effecting an estimated saving in cost of construction and real estate of more than $10,000,000. This new plan was endorsed by all concerned, including the writer, and the tunnel was built according to it.

Although the through route from Long Island to New Jersey has been slow in emerging in complete form, it is gradually developing step by step. Another section of highway about three miles long, to be built from the easterly end of the present highway extending from the Queens plaza of the Queens Midtown Tunnel, is included in New York City's postwar program. It was recognized from the start that each link should form a part in a through express highway, and the links so far built or authorized fit into this scheme.

The writer is in accord with the plans of all sections so far built or authorized, with one exception. The New Jersey approach to the Hudson River tunnel, involving on the main route a corkscrew spiral with a continuous lift of 268 ft from under the river to the top of the Palisades, coupled with curves having radii of less than 300 ft for as much as one half of the complete circular loop, is not up to the standard of modern highway requirements, especially for commercial traffic.

A more detailed description of the various elements involved in the complete east-west route (of which the Queens Midtown Tunnel is a link) and their development is given in the writer's more complete paper filed in the Engineering Societies library at New York.[6a]

It is true, as Mr. Conover states, that the success of a vehicular tunnel depends largely on the plaza layouts together with the approaches.

The suggestion that the tunnel avenues for the Mid-Manhattan Underpass should correspond, for exit and entrance traffic flow, to the new avenues for the Queens Midtown Tunnel was given considerable study. The conclusion reached was that this arrangement would be expensive and impracticable. Although it

[6a] 29 West 39th St., New York, N. Y.

would provide a slightly simpler traffic flow at some intersections, the disadvantages outweighed the advantages.

Automatic control of ventilation for a large tunnel, such as the Queens Midtown, would be an added expense with no saving or advantage gained, because an operator would have to be on duty at the remote control buttons at all times. Peaks of short duration would result in a great deal of fan-speed shifting if automatically controlled. These peaks can be "ironed out" with manual operation. The Oakland-Alameda Tunnel has (or had) automatic control. Automatic control is deemed advisable only on short and less important tunnels where such operation actually will save manpower on the operating payroll.

City fire apparatus has often passed through the Queens Midtown Tunnel, not on fire duty, but for access to the repair shop in Long Island City. No difficulty has been experienced on account of clearances. One lane only is necessary for the city's longest hook-and-ladder equipment, even on the smallest radius curve near the Manhattan portal.

Dr. Levy discusses, in considerable detail, the precautions taken during the construction of the Queens Midtown Tunnel to protect the men against compressed air illness, and also certain experiments which were undertaken, having in mind the development of better protection of the men from a medical point of view.

As one result of the investigations made on helium-oxygen mixtures, the installation and use of apparatus for the alleviation of ear block is specified as a requirement on the compressed air work on the Brooklyn-Battery Tunnel, on which work is now (1944) temporarily suspended on account of war conditions. If the results prove to be as satisfactory as anticipated, it may well be that the installation and use of apparatus to alleviate ear block should be a standard requirement for future compressed air tunnel work.

On the basis of past experience, and especially the experience on the Queens Midtown Tunnel, Dr. Levy suggests further study in regard to the present requirement of the New York State rules for the installation of a second tunnel bulkhead when pressures exceed 21 lb per sq in. The double bulkhead method of decompression did not show any advantage over the use of the single bulkhead on the Queens Midtown Tunnel in so far as the incidence of "bends" cases is concerned, and it seems appropriate to suggest that the present regulations might be modified advantageously to require that a second bulkhead be used as required by considerations of safety depending upon the physical conditions rather than upon any given air pressure in the tunnel heading. On the Manhattan side of the Queens Midtown Tunnel, where the distance between bulkheads was approximately 430 ft in one tunnel and 470 ft in the other, the second bulkheads were placed primarily because of the requirement of the regulations. As the headings entered full rock section in the rock reef approximately only 1,000 ft from the first bulkhead, serious consideration could have been given to the omission of the second bulkhead had it not been for the code requirement.

The question as to whether, in decompressing, the full amount of the fast initial drop of pressure as required by the present regulations may or may not be

beneficial, seems to the writer to be a matter for further experimentation. It appears that Dr. Levy makes a pertinent statement in this connection when he writes, "In fact, it well may be that the fast initial drop of pressure in this method of decompression actually induces the beginning of the formation of the air bubbles that are the cause of compressed air illness."

Dr. Levy also suggests that further study be given to working one shift of six hours rather than two shifts of three hours in low pressures up to 18 lb per sq in. and possibly higher. Although this would be an interesting experiment, there may be some practical objections to it in actual practice. It may be questionable whether a man working six hours continuously, regardless of whether or not he is working in compressed air, would produce a satisfactory six hours' work. There is a present-day tendency in certain industries to give employees short rest periods at various intervals during the day to offset fatigue. It may be questioned whether a man working six hours continuously would be operating at full efficiency during the latter part of the period, and this condition might be hazardous in tunnel work where alertness and efficiency are required at all times for reasons of safety.

Professor Legget has raised several pertinent questions: The ratchet wrenches used for bolt tightening were standard equipment. Fixed maximum-torque wrenches have been considered for use on the Brooklyn-Battery Tunnel, where the minimum bolt stress specified is 30,000 lb per sq in. It was found that there were no maximum-torque wrenches on the market of sufficient capacity for this specification and one was ordered to be constructed specially. It was impossible to get this wrench on account of war conditions. If the trial had proved successful it was contemplated that inspectors of the Tunnel Authority would be provided with torque wrenches and would spot-check the bolts for tightness. It was considered impracticable to use torque wrenches for the actual tightening.

In experimenting to determine the torque necessary to produce a given stress, it was observed that there was quite a range in the frictional behavior of different combinations of bolt, nut, and washer. This would be expected, especially since the bolts are forged and the threads rolled. In this process threads are sometimes burred in packaging and handling, and the nuts and washers have a relatively rough finish.

When conditions permit resumption of construction work on the Brooklyn-Battery Tunnel, the investigation of the use of torque wrenches to test bolts will be continued.

The drainage system considered for the Manhattan plaza of the Queens Midtown Tunnel involved excavating to expose the rock surface over the entire plaza area, the construction of a fairly complicated system of drainage pipes, galleries, sumps and pump chambers, and backfilling to subgrade of the invert slab with gravel. The heavy slab was estimated to cost little more than the drained plaza, which involved more excavation, the cost of gravel backfill, and higher unit costs of concrete than the massive slab, besides the drainage system itself. As the cost was practically the same as that for the heavy slab design and there was an unusual ground-water condition (ground water being as high as 19 ft above mean high water at one point) the heavy invert slab was adopted as involving no operation and maintenance costs.

Clay for blanket purposes is chosen because it is a dense, erosion-resistant material that can be obtained economically in the large quantities required. This generally results in a clay being obtained from a near-by source. The clay used for blanket purposes on the Queens Midtown Tunnel had a weight of 115 lb per cu ft and was dredged from Newark Bay, a previously used source of clay for blanket purposes.

The volcanic clay referred to is a form of bentonite described as a type of clay originating from volcanic eruptions and consisting chiefly of mineral montmorillonite, a hydrous silicate of aluminum. It is found almost exclusively in the Black Hills region of the United States (South Dakota). It possesses in high degree the qualities of absorption and swelling, and will re-swell after drying for an indefinite number of cycles. Its swelling properties are used in retarding or stopping the flow of water, and in the tunnel it was used as an aid in the attempt to reduce the escape of compressed air through porous material.

Mr. Brodie's analysis of the tunnel ring, together with the resulting design for the cast-iron lining of the Holland Tunnel which he describes, is still representative of fundamental principles in this art. Very important is his statement, "Subaqueous shield-tunnel practice recognizes the uncertainty of absolute values of external loading, especially at critical times during construction." There is sometimes a tendency for theorists not familiar with practical shield tunneling to lose sight of this principle.

The lining section actually used for the Holland Tunnel is not to be confused with Fig. 18, "Cast-Iron Details, Heavy, 29-Ft Sections, Holland Tunnel." The outside radius of the adopted section was 14 ft 9 in. and in some cases 15 ft 2 in.; the web thickness was $1\frac{5}{8}$ in. increasing to $2\frac{3}{4}$ in. at the flanges. Mr. Brodie made it clear in his discussion that the lining shown in Fig. 18 formed the basis of the design and analysis but that the modified sections, with the foregoing dimensions, were actually used. The figure alone, however, might be misleading.

Referring to feature (8) of Mr. Brodie's discussion, "effect of transverse tie rods," it may be pointed out that the firm character of the material surrounding the Queens Midtown Tunnel tubes made such permanent tie rods unnecessary, except for the short stretches under the Manhattan bulkhead and at the midriver sump, as stated in the paper.

In his letter of September 27, 1943, to the writer, written only eight days before his death, Mr. Brodie requested that in this discussion mention be made of those who assisted in his work on the design of the cast-iron lining for the Holland Tunnel, as follows: Henry G. Babcock, Assoc. M. Am. Soc. C. E., James H. Dugan, M. Am. Soc. C. E., Jacob Mechanic, Assoc. M. Am. Soc. C. E., and Morris Cohen.

Some of the statements made by Mr. Eremin call for further comments, and a few of them will be discussed briefly. The protective blanket acts primarily as a counterweight against rupture of the river bed due to the unbalanced air pressure in the upper part of the face of the tunnel heading. If the face of the tunnel is in porous material, the imperviousness of the blanket does not accomplish much in preventing the escape of air from the tunnel. As escaping air from the tunnel does find fissures and other such avenues of escape through

the blanket, the blanket is eroded to an extent sometimes requiring constant replacement under these circumstances. If the blanket were entirely impervious, air would continue to escape from a tunnel heading in porous material and find its way out around the edges of the blanket. A dense and erosion-resistant material is indicated for blanket purposes.

If the tunnel heading is in impervious material, the required air pressure is generally less than theoretical hydrostatic head and there is no unbalanced air pressure tending to blow out the river bed. Under these conditions a blanket would not be necessary, even though the natural cover over the tunnel were not great. For this reason blankets and loss of air in shield tunneling are generally associated. To the best of the writer's information, the blanket used in the East Boston Tunnel was used to build up low spots in the river bed, thus providing a uniform total thickness of cover of only 15 ft over the tunnel. Altogether only 7,200 cu yd of clay were used for the blanket, most of it deposited at pierhead lines to consolidate the mud and increase cover from 9 or 10 ft to 15 ft, as a precaution against material running into the heading. There was no danger of a blowout as the material penetrated by the tunnel was largely impervious clay, plastic to stiff in consistency, and air pressures used were generally below theoretical hydrostatic head.

The use of grout to reduce air leakage at the face, and its use to fill the annular space around the tunnel lining left by the tail of the shield should not be confused. Its latter use is standard practice and has little, if any, effect on shield progress, whereas the effectiveness of grouting the face, which is done only under unusual circumstances, might have some effect on shield progress.

Mr. Eremin should not include the Lincoln Tunnel with his examples of structural steel-lined shield-driven tunnels. The shield-driven sections of the Lincoln Tunnel were lined with cast iron; the approach tunnels in full section of rock, built without the use of shields, were lined with steel, as were the Manhattan approach tunnels of the Queens Midtown Tunnel.

The writer had no intention of inferring that further study is not necessary in the planning and designing of tunnels for highway traffic. On the contrary, he is convinced that each such project requires a large amount of study and planning to meet the conditions peculiar to each location. The writer had in mind, however, that only twenty-five years ago (1919) long tunnels, designed for the exclusive use of motor-driven vehicles, were considered in the nature of an experiment and were looked upon with skepticism, not only by the general public but also by some engineers. During the intervening period extensive research work has been conducted on some of the important phases relating to this kind of project and much has been learned through operating experience of the several such tunnels which have been built and are now in operation.

When the Holland Tunnel was being planned, an extensive program of research was conducted, involving the expenditure by the Tunnel Commissions of more than $100,000 and by the cooperating agency, the U. S. Bureau of Mines, of a considerable amount. This research work was intensively promoted over a period of two years and since has been continued intermittently to provide supplementary information on related phases of the same problem. The uncertainty and apprehension with regard to this phase of vehicular tunnel

design were effectively overcome by the striking success of the ventilation system developed as a result of that research work.

Another illustration of a question that was then considered uncertain by some engineers was the suitability of cast iron as the primary lining for such a large tunnel in a material like the Hudson River silt. The design of the lining for the Holland Tunnel, which involved a much larger diameter in Hudson River silt than had before been constructed, was quite properly the subject of open technical discussion. In one paper[27] the opinion was expressed that "In soils with a low angle of repose, for any specific case presented, there is a limiting inside diameter (which may be determined for that case) beyond which the primary lining, whether of cast iron or of concrete blocks, cannot be designed to stand without internal bracing." At considerable variance with the design and analyses of the Holland Tunnel engineers, and predicated on the line of reasoning on which the foregoing conclusion was based, it was determined in that paper that the inside diameter of a cast-iron lined tunnel not subject to nonelastic deformation in Hudson River silt would be limited to 26.2 ft with flanges more than 5 ft deep and an outside diameter of more than 36 ft, the lining segments at this diameter being unwieldy in size, number, and weight. However, with faith in their own determinations, the Holland Tunnel engineers designed and successfully constructed in Hudson River silt a cast-iron lining of 29 ft 6 in. outside diameter and 14-in. flanges or 27 ft 2 in. inside diameter, the general features and details of which have subsequently set the pattern for cast-iron lining of 31 ft outside diameter in both the Hudson and East rivers. A synopsis of this design and analysis is mentioned in Mr. Brodie's discussion.

One more illustration of the extensive research work which was necessary heretofore is that referred to in the comments on Mr. Thoresen's discussion regarding the development of a suitable wall tile for this type of tunnel.

The writer is indebted to Mr. Tuttle for a concise statement of the historical background of the Queens Midtown Tunnel.

The adequate highway approaches to the tunnel, both in Manhattan and Queens, to which Mr. Tuttle refers, have to a large extent yet to be provided. That these approaches could not have been included, as part of the tunnel project on a self-liquidating and self-supporting basis, is self-evident. It is generally the practice of New York City to make its toll river crossings a real part of its free highway system. In the case of the Queens Midtown Tunnel this practice has met with serious delays which the writer hopes will soon be overcome.

[27] "Notes on Tunnel Lining for Soft Ground," by S. Johannesson and B. H. M. Hewett, *Transactions* Am. Soc. C. E., Vol. LXXXIII (1919–1920), pp. 1845 and 1855.

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DISCUSSIONS

SEDIMENTATION IN RESERVOIRS

Discussion

BY A. L. SONDEREGGER

A. L. SONDEREGGER,[12] M. AM. SOC. C. E.[12a]—The accelerated rate of reservoir construction in the United States since about 1910, together with the marked loss of storage space within a comparatively short period in many of these structures, has focused the attention of engineers on the problem of reservoir sedimentation. Two thirds of the thirty-two papers in the Bibliography of this paper were published since 1938. Kindred subjects were discussed at the July, 1943, meeting of the Society in Los Angeles, Calif., by the U. S. Geological Survey (57),[12b] and in recent special reports issued by the Soil Conservation Service.

The origin and nature of sediments have been described in this paper and in other papers, and records of the rate of debris deposition are available for a large number of representative watersheds. Hence, it is possible to forecast with some degree of probability the rate of sedimentation which may take place in structures, directly intercepting flood waters. However, a reconnaissance of physiographical and geological conditions and of the effect of the invasion of the watershed will be required.

Under the heading, "Remedies for Reservoir Silting," the author correctly states that authorities generally agree that redemption of lost reservoir capacity is practically impossible. In this connection, he mentions the removal of debris in Southern California by mechanical excavation, a process feasible with relatively shallow reservoirs in open terrain; the removal of silt deposits by sluicing which has been practiced with partial success; and last, but not least, the utilization of density flows. Since full redemption of lost capacity is not feasible, the question then arises as to what extent sedimentation may be prevented or controlled.

NOTE.—This paper by Berard J. Witzig was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Joe W. Johnson; October, 1943, by Messrs. John W. Stanley, Stafford C. Happ, and Thomas H. Means; November, 1943, by Carl B. Brown, and C. S. Jarvis; and December, 1943, by Hugh Stevens Bell, and Harry F. Blaney.

[12] Cons. Engr., Los Angeles, Calif.

[12a] Received by the Secretary January 31, 1944.

[12b] Numerals in parentheses, thus: (57), refer to corresponding items in the Bibliography (see Appendix II in the paper), and at the end of discussion in this issue.

The basic causes of debris production are the disintegration and decomposition of rocks. These processes are as old as the hills, and nothing can prevent them; nor can the removal of debris by periodically occurring storms and concomitant runoff be stopped. Production of debris and erosion are at a minimum when a watershed is protected by its natural vegetal cover. Under such conditions a natural physiographic balance is maintained, and probably attempts by man to improve on these conditions would not be successful. Nature apparently is operating under the principle of self-conservation which automatically adjusts itself to varying conditions, climatic and physiographic (58).

Invasion of the watershed; effects of deforestation, overgrazing, cultivation, unstable slopes in road cuts; and other methods have generally tended to upset nature's balance and to increase sediment production and removal. The writer is impressed with the benefits that ultimately will result from the recent practice of erosion control initiated by the Soil Conservation Service to correct the destructive effects of land abuse. Many of the methods designed to restore the vegetal cover will become fully effective only after persistent efforts over periods of years. However, the long-range results should fully justify the expenditure.

The author concludes (see "Remedies for Reservoir Silting") that "the most certain means of maintaining reservoir capacity is to prevent sediment from forming, or, after formation, to prevent its deposition in the lake." Although this theory is unquestionably correct, generally, it is not practicable. In this connection, the author has quoted Messrs. Eakin and Brown (1) as follows:

> "Main reliance for material and permanent conservation of reservoir resources must rest upon control of sediment production at primary sources through more widespread and effective application of established methods of erosion control."

This statement may apply to the disturbed watershed, but hardly to natural conditions. Since debris production cannot be reduced below its natural minimum, the preservation of water storage must rely mainly on the creation of separate space for sediment storage.

In many storage reservoirs, debris space has been created by raising the height of dams. On the other hand, storage and detention projects in large numbers have been constructed on silt-carrying streams without any provision whatever for preserving the integrity of the hydraulic function. The results have been little short of disastrous in many cases, not only because of the loss of the investment, but also because of the scarcity of reservoir sites.

In any storage project where sedimentation is a factor, provision for the long-time disposal of debris should be made part of the original project and should be included in estimates of costs and in budgets. The plans may contemplate additional height of dam, "upstream engineering" by the construction of a series of permanent debris barriers, provision for deposit grounds for sediments eventually to be removed, or other methods.

The author states (see heading, "Introduction") that "Studies by the Soil Conservation Service (SCS) of the U. S. Department of Agriculture (1) show that, in the silt-carrying streams of the United States, the probable life expectancy of existing reservoirs is dangerously short" and "* * * that about 64% of all reservoirs have useful lives of less than 100 years, * * *." Thus, a reduction of 80% in the capacity of a reservoir is assumed to terminate its useful life.

On the basis of these findings, the broad rule might be established that, in order to safeguard the storage supply of a community in the semiarid regions for 100 years or more, debris storage must be provided in volume equal to, or exceeding, the water storage. This is a "large order" and may be hard to fill; but it epitomizes the seriousness of the problem.

Although such provisions are not remedies, they tend to postpone the day of reckoning. Acting in accordance with the best available knowledge, engineers at least will escape the condemnation that was heaped on Louis XV when he shrugged his shoulder, saying "After us the deluge." It might be well to recall the fate that befell his grandson barely 35 years later during the French Revolution. The wise man has said that history repeats itself.

Coming generations may see more clearly and deeply into the true nature of things and find the solution that will save them from the fate suffered by some of the older civilizations in the semiarid regions.

The author is to be commended for the presentation of an excellent and condensed review of a complex problem.

Bibliography.—

(1) **"Silting of Reservoirs,"** by Henry M. Eakin (revised by Carl B. Brown), *Technical Bulletin No. 524*, U. S. D. A., 1939. (*a*) Fig. 19, facing p. 98. (*b*) Table 13, pp. 120–123.

(57) *Water Supply Paper No. 844*, U. S. Geological Survey, 1942.

(58) **"Modifying the Physiographical Balance by Conservation Measures,"** by A. L. Sonderegger, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 285.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

A SYMPOSIUM

Discussion

BY LOUIS BALOG, WILLIAM G. RAPP, AND LEON BESKIN

LOUIS BALOG,[38] ESQ.[38a]—The papers on the Rainbow Arch Bridge record considerable progress in the design and construction of long-span arch bridges. The designers of the bridge present, in the first paper, a method of analyzing flexible arches that yields correct results, and they also derive practical design formulas. The results of the experimental investigations given in the second paper check the designers' analysis and are useful for preliminary design of flexible two-hinged arch ribs. In Appendix I of the first paper, methods for the consideration of deflection effects are discussed. In addition to those mentioned a method developed by Fr. Dischinger[39] is of interest. It applies to beam-columns and arch ribs, primarily those having a parabolic axis and a varying cross section expressed by

$$I_c = I_x \cos \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

The increase of the elastic-theory moments is expressed by a rapidly converging geometric series:

$$M = M_E \left(\frac{\nu + \beta}{\nu - 1} + \frac{\beta_1}{\nu} + \frac{\beta_2}{\nu^2} + \frac{\beta_3}{\nu^3} + \cdots \right) \ldots\ldots\ldots\ldots (57)$$

in which (in addition to the notation of the Symposium): β is a numerical coefficient; and

$$\nu = n \frac{E I}{H a^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

NOTE.—This Symposium was published in October, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1943, by Egidio O. Di Genova, and Charles Mackintosh; January, 1944, by C. M. Goodrich, and T. Kennard Thomson; and February, 1944, by C. M. Spofford, L. J. Mensch, and O. H. Ammann.

[38] Cons. Engr., New York, N. Y.

[38a] Received by the Secretary January 25, 1944.

[39] *Der Bauingenieur*, Vol. 18, August 20, 1937, pp. 487–520.

In Eq. 58: a is half the span length; and n is a numerical buckling coefficient.

For ribs that are not too flexible the consideration of the first term in the parentheses at the right side of Eq. 57 is sufficient to estimate the increase of the elastic-theory moments. For a hingeless rib the following approximate formulas are obtained for the half span uniformly loaded:

Moment at springing—

$$M_S \approx M_E \left(\frac{\nu - 0.382}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (59a)$$

moment at the quarter point—

$$M_{0.25} \approx M_E \left(\frac{\nu - 0.014}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (59b)$$

and, moment at the three-eighth point—

$$M_{0.375} \approx M_E \left(\frac{\nu + 0.121}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (59c)$$

For use in Eqs. 59 (see Eq. 58), $n = 20.19$. Introducing the data given in Table 3 for the hingeless rib: $\nu = 20.19 \dfrac{29{,}400 \times 88.2 \times 144}{6{,}460 \times 475^2} = 5.18$; and, by Eqs. 59*a*, 59*b*, and 59*c*, respectively, $M_S = 1.148\, M_E$; $M_{0.25} = 1.236\, M_E$; and $M_{0.375} = 1.268\, M_E$. These approximate values are from 2% to 6% different from the dead-load plus live-load moment values given in Fig. 12 and Table 3, indicating the correctness of the moments used in the design.

For a two-hinged rib the approximate moment at the quarter point, when the half span is loaded, is

$$M_{0.25} \approx M_E \left(\frac{\nu + 0.032}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

in which (see Eq. 58), $n = 9.87$. Introducing the data given for the two-hinged rib in Table 2: $\nu = 9.87 \dfrac{29{,}400 \times 120.24 \times 144}{7{,}551 \times 475^2} = 2.95$; and substituting in Eq. 60: $M_{0.25} = 1.53\, M_E$. This value is 10% less than the correct value of the dead-load plus live-load moment given in Table 2. On account of the flexibility of the two-hinged rib the approximation is not as close as that for the more rigid hingeless rib.

By presenting comparative data for the two-hinged arch and for the hingeless arch, the designers have directed attention to the superiority of the more statically indeterminate structure. The hingeless rib of greater rigidity was designed with considerable saving in metal. The efficient section of this rib resulting from the use of stiffened webs of minimum section should also be noted.

The study of the four papers on the Rainbow Arch Bridge reveals the care and understanding with which this outstanding structure was designed and built.

WILLIAM G. RAPP,[40] Assoc. M. Am. Soc. C. E.[40a]—In its entirety this Symposium is an excellent contribution to the limited existing literature on design, engineering, fabrication, and erection of monumental structures such as the Rainbow Arch Bridge. In particular, a distinct service has been rendered to that part of the engineering profession engaged in the actual prosecution of work in the field—to those engineers on whose shoulders falls the duty of seeing that the structure as designed and fabricated is erected properly, safely, and efficiently. Such practicing engineers must meet unexpected and vital problems with immediate answers, an art that can only be achieved by actual, tedious, and arduous experience, or by the study of papers such as that portion of the third Symposium paper dealing with "Erection Controls" and the entire fourth paper, "Erection of Steel Superstructure."

To the designer, the design of a bridge of this caliber is often an important and seemingly difficult task. Decisions as to type, sizes, etc., are necessary before any further steps can be taken. The engineering work following the basic design decisions is just as important and often just as difficult. Fabrication, similarly, presents important problems, although these can generally be overcome.

The final and most difficult decisions of all, however, are those which must be made by the erector: By the office engineers, who must prepare complete plans for the subsequent erection of the structure, and by the resident engineer, together with all those associated with him at the site. In the preliminary office study: Methods, equipment, stresses and strains in the structure, scheduling, planning and foreseeing all the minor and major details of erection from start to finish; and, for the resident engineer: Unexpected terrain, unforeseen weather, unpredictable working personnel, or difficult field connections—all these problems must be mastered or the design, the engineering, and the fabrication will have been wasted. The structure must be built and too little help has been given, in the past, toward disseminating theoretical and practical information—studies, analyses, and records of actual construction at the site. The more aid that can be given in this phase of engineering, the better the finished project will reflect the original ideas of the designing engineer. Also, the more publicity given to actual field erection problems or difficulties, the better the designing, detailing, and fabricating engineers will be able to solve these problems, by preventing the conditions which too often exist as a result of lack of information on this phase of construction.

LEON BESKIN,[41] Assoc. M. Am. Soc. C. E.[41a]—The purpose of the following comment is to correct some errors in the first two papers of the Symposium and to show that very simple formulas are available for design and even for definite analysis of flexible arches. The results thus obtained give the same degree of accuracy as the lengthy calculations indicated in the Symposium, and make tests useless.

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[40] With Bethlehem Steel Co., Erection Dept., Bethlehem, Pa.

[40a] Received by the Secretary February 14, 1944.

[41] Senior Stress Analyst, Consolidated Vultee Aircraft Corp., Allentown, Pa.

[41a] Received by the Secretary November 23, 1943.

Deflection Effects and Design Formulas.—It is regrettable that the expression "elastic theory" has been used in the first Symposium paper to indicate a method of stress analysis in which deflections are supposed to be infinitely small. This expression is certainly a misnomer according to all conventional definitions (see A. E. H. Love[42]). S. Timoshenko studies the "deflection theory," in his "Theory of Elastic Stability."[43] The expression "deflection theory" seems acceptable, but as a subdivision of the elastic theory, and not in opposition to it; the expression "beam-columns" is also widely used.

The statement that several deflection theories are available for arch ribs throws an incorrect light on the problem. Written in differential terms, the problem is elucidated completely, and the theory is unique. Methods of integrating the differential equation may differ, generally because terms are neglected in order to simplify the calculations. The fact that results may differ shows that simplifications made are not justified and lead to inaccurate or erroneous results.

Under the action of axial forces and lateral loads curved beams deflect differently from beams submitted to lateral loads only. Many years ago, the late Milo S. Ketchum,[44] Hon. M. Am. Soc. C. E., presented a formula for the combined effects of bending and axial loads in the elastic range. Some formulas developed subsequently herein will show the simplicity of the problem, even if a fairly close approximation (say, within 1%, generally) is required. They avoid unnecessary loss of time in design and they demonstrate the effect of deflections on stresses as soon as the working stress under axial load, the span, the depth and type of beam, and the end conditions are selected. From those formulas a "factor of amplification," c, can be determined which represents the ratio of actual deflections to deflections calculated for a beam without axial load, and also the ratio of moments in both conditions. That factor is defined as

$$c = \frac{1}{1 + \frac{s_a (L_1)^2}{\pi^2 E k^2}} = \frac{1}{1 - \frac{T}{T_c}} = \frac{1}{1 + \frac{T (L_1)^2}{\pi^2 E I}} \quad \ldots\ldots\ldots\ldots (61)$$

in which, in addition to the notation of the Symposium: s_a = unit stress due to axial load; k = radius of gyration of the arch; T_c = thrust that causes buckling; and L_1 = effective free length of span. The latter is the total length for a hinged beam under an applied active axial load. It is also the length of the arc between the crown and the skewbacks for a hinged arch under the action of applied transverse loads applied on half of the span and thrust or 0.7 of that length, approximately, if the ends are fixed.

Definition of the Factor of Safety N by Eq. 1.—The persistent use of factors of safety that differ for each type of load is regrettable. Reducing the factor of safety for wind and temperature to 1 is equivalent to reducing those loads in the ratio of the factor of safety used for the other loads. As wind and temperature loads are somewhat arbitrary, the use of a variable factor of safety is a useless

[42] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, 4th Ed., Cambridge Univ. Press, 1927, p. 92, l. 15.

[43] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., 1936.

[44] "Structural Engineer's Handbook," 2d Ed., McGraw-Hill Book Co., Inc., New York, 1918, Section 38, p. 57.

complication, especially when various factors of safety are used for different combinations of loads.

Equations of Equilibrium for Curved Beams.—The factor $\frac{T}{r_o E A}$ in Eq. 2 is obviously erroneous, and must be omitted, since under uniform axial stress, with necessary radial loads to maintain it in equilibrium, the radius of the ring does not change. On the contrary, if there is a strain e_1 along the radius, the latter changes by $e_1 r_o$.

If stresses along the radius are negligible, the quantity e_1 is equal to $-\frac{\mu T}{E A}$ (in which μ = Poisson's ratio), and the corresponding factor is $+\frac{\mu T}{r_o E A}$. If the terms due to shear are taken into account, Eq. 2 must be written:

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E I} - \frac{1}{E_s A_w}\left(W_n + \frac{T}{r_o}\right) + \frac{\mu T}{E A r_o} \quad \ldots\ldots (62)$$

in which W_n is the component of the load normal to the arc, per unit length of arc.

Eqs. 3 are obviously incorrect because the tangential component v of the deflection is omitted. That component is not negligible, since it produces secondary bending in a two-hinged arch when the moment M'_a is proportional to y, and no camber for dead load is provided.

The term $\frac{u}{(r_o)^2}$ cannot be neglected in a refined analysis (see the comparison between the "exact" procedure and the approximate method presented by Mr. Hardesty, et al.) Comparing the value of $\frac{u}{(r_o)^2}$ to that of $\frac{M}{E I}$ in the nondeflection theory, the ratio is

$$\frac{u:(r_o)^2}{M:(E I)} = \frac{12 \times 14:(827)^2}{23{,}000:(29{,}400 \times 120)} = 0.036 \quad \ldots\ldots (63)$$

A direct estimate of that ratio can be made, assuming that the moment M and the deflection u are sinusoidal functions along a circular arch, of the type $\frac{R_{td} \sin \alpha \pi}{\alpha_s}$. Then,

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = \frac{1}{(r_o)^2}\left(\frac{d^2u}{d\alpha^2} + u\right) = \frac{R_{td}}{(r_o)^2}\frac{d^2u}{d\alpha^2}\left[1 - \frac{(\alpha_s)^2}{\pi^2}\right] \quad \ldots\ldots (64)$$

in which R_{td} is the ratio of thrust to dead load; and α_s is the slope angle at the skewback. Thus, the relative influence of the second term is $\frac{(\alpha_s)^2}{\pi^2}$, which has the value 0.038 for $\alpha_s = 35°$.

Both methods demonstrate that an error of 4% occurs in the value of the moment, when the term $\frac{u}{(r_o)^2}$ is neglected. When the amplification due to deflection is introduced (as shown subsequently), the error, which is unconservative, is increased to 6%; thus, the "exact method" applied by Mr. Hardesty, et al., is no more accurate than the approximate method.

The secondary moments due to the effect of change in length of the arch under dead load are eliminated by the introduction of a camber. The moments thus eliminated are actually of the same order of magnitude as the moments omitted with the factor $\frac{u}{(r_o)^2}$. The exact value of the horizontal component of the thrust is:

$$H = \frac{\int y\, M'_a\, ds{:}(E\,I) - \int \sin\alpha\, dM'_a{:}(E\,A) + \int \sin\alpha\, dM'_a{:}(E_s\,A_w)}{\int y^2\, ds{:}(E\,I) + \int \cos^2\alpha\, ds{:}(E\,A) + \int \sin^2\alpha\, ds{:}(E_s\,A_w)} \quad \ldots(65)$$

Assuming that the ordinates of the arch are proportional to the curve of moment due to dead load, $M'_a = H_o\, y$, and, with an error of the second order of magnitude, assuming a constant cross section:

$$\frac{H}{H_o} = 1 - \frac{\int dx{:}(A\,E)}{\int y^2\, dx{:}(I\,E)} = 1 - \frac{15\,k^2}{h^2} \quad \ldots\ldots\ldots\ldots(66)$$

The difference between H and H_o produces moments in the arch, which are equal to

$$M = M'_a - H\,y = \frac{M'_a \times 15\,k^2}{h^2} \quad \ldots\ldots\ldots\ldots(67)$$

For the arch investigated (approximately): $k^2 = 28$; $h = 150$; and, at the crown, $M'_a = \frac{9.5 \times 950^2}{8} = 1{,}050{,}000$. Thus, $M = 2{,}500$. At the quarter point, the value of M is approximately 1,900, which represents 5% of the total moment at that point, and is somewhat smaller than the error of 6% made when neglecting the factor $\frac{u}{(r_o)^2}$. This shows the regrettable fact that, although the authors have been careful to eliminate some secondary bending by special erection methods, moments of equal importance, due to other factors, have been completely omitted from the analysis.

The methods of successive approximation used by the authors to account for the effect of deflections can be simplified by direct integration of the equations of equilibrium, as this is generally done in problems of beam columns. Using Eqs. 5 and 6 (in which the product $\Delta H\,\eta$ is of second order and can be omitted), and substituting the approximations, $\Delta H = 0$, $v \sin\alpha_s = 0$, the equation of equilibrium is:

$$-\frac{E\,I\,d^2u}{ds^2} = M_{ae} + T\,u \quad \ldots\ldots\ldots\ldots(68)$$

As a first approximation, the moment M_{ae} due to live load is given by a sinusoidal line. Then the integration is immediate, and with $M_{ae} = \frac{M_o \sin 2\,\pi\,s}{L'}$:

$$E\,I\,u = \frac{M_o \sin 2\,\pi\,s{:}L'}{1 + \frac{T\,(L')^2}{4\,\pi^2\,E\,I}} \quad \ldots\ldots\ldots\ldots(69)$$

Thus,

$$M_{af} = \frac{M_{ae}}{1 + \frac{T\,(L')^2}{4\,\pi^2\,E\,I}} = c\,M_{ae} \quad \ldots\ldots\ldots\ldots\ldots\ldots (70)$$

Eq. 70 explains why "it was noted that the variation in Δm_1 * * * was similar to the variation in M_e * * *" (see paragraph containing Eq. 8). Furthermore, the value of the quantity $\frac{\Delta m_{a1}}{M_{ae}} = \frac{T_a\,u_{ae}}{M_{ae}}$ is thus determined. Eq. 13 contradicts the fact that M_{ae} and Δm_{ae} are proportional; an average value of T must be used in Eqs. 13 and 14, instead of a variable value.

The coefficient of amplification c (Eq. 61) gives accurate results for a final analysis. For instance with: $T = 7{,}588$; $M = 24{,}950$; $E = 29{,}400 \times 144$; $I = 120$; and $L' = 1{,}020$; Eq. 37 yields $c = 1.64$ and $M = 40{,}900$. This value of M differs by 150 (0.4%) from the value calculated by the authors. If the correction for $\frac{u}{(r_o)^2}$ is introduced, the amplification coefficient is

$$c' = \frac{1}{1 + \frac{(L')^2}{4\,\pi^2}\left[\frac{T}{E\,I} - \frac{1}{(r_o)^2}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots (71)$$

Thus the moments are increased by 6% above the values calculated by the authors. This clearly shows that the so-called "approximate methods" are quite sufficient for practical purposes. Otherwise all terms must be taken into account and the computations will be complicated by the need for solving a differential equation of second order. To avoid it, an error of 6% is introduced; in view of that error, all the refinements introduced by Mr. Hardesty, et al., become meaningless.

A method of integration that is always powerful for problems of this type involves the use of developments in trigonometric series. M. Hupner[45] has developed the theory of buckling in hinged and fixed arches and presents general formulas for the amplification of deflections. Values of Δm_a and M_{ae} are proportional when the curve of M_{ae} is a sinusoid. This property is only approximate for other laws of variation of M_{ae}. When $M_{ae} = M_o\left(\frac{\sin 2\,\pi\,s}{L'} + \lambda\,\frac{\sin 4\,\pi\,s}{L'}\right)$ (in which λ is a constant), the amplification factor at the crown is:

$$c_1 = \frac{c}{1 + \lambda}\left[1 + \lambda\,\frac{\pi^2\,E\,I + T\,(L')^2{:}4}{\pi^2\,E\,I + T\,(L')^2{:}16}\right] \quad \ldots\ldots\ldots\ldots (72)$$

For $\lambda = \pm\,0.1$ and $c = 1.64$, the values of c_1 are, respectively, 1.59 and 1.70. If the moment M_{ae} is constant on each half span, with opposite values on the two halves, the factor of amplification is 1.80, corresponding to $c = 1.64$; for a parabolic distribution of moments, it is 1.658, which shows that a sinusoid gives a very accurate value of the coefficient of amplification for that case.

[45] "Application des Séries Trigonométriques à l'étude du flambement des arcs," par M. Hupner, *Annales des Ponts et Chaussées*, November, 1935, p. 754.

In reference to Eq. 16, the following can be noted:

(*a*) An average value of T must be used instead of T_a, in the denominator.

(*b*) That average value must include the thrust due to wind and temperature.

(*c*) There is no reason to omit the coefficient N for wind and temperature effects.

(*d*) The separation of moments into those due to dead load and those due to live load for the application of the coefficient of amplification is theoretically incorrect but is certainly acceptable. Theoretically the moments must be developed into components to which various coefficients of amplification must be applied:

$$s = \frac{T_a}{A_a} + \frac{M_{1a}}{S_a} + \frac{M_{2a}}{S_a\left(1 - \frac{T}{T_c}\right)} + \frac{M_{3a}}{S_a\left(1 - \frac{T}{4\,T_c}\right)} + \cdots \quad \ldots\ldots(73)$$

In Eq. 73 the buckling value of the average thrust T_c is approximately equal to $T_c = \frac{4\,\pi^2\,E\,I}{(L')^2}$.

A more exact method of computation is indicated subsequently herein (see Eq. 83). For practical purposes all terms after the third, in Eq. 73, can be omitted, and the total moment M can be divided into a part that is not affected by the coefficient of amplification, and a part that is affected by that coefficient. This shows that an expression of the type:

$$s = \frac{T_a}{A} + \frac{M_{1a}}{S_a} + \frac{M_{2a}}{S_a\,(1 - T{:}T_c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(74)$$

seems acceptable.

The condition $N\,T < T_c$; or, as written in the Symposium (see paragraph following Eq. 16), $N\,T_{0.25}\left(\frac{u_{0.25\,E}}{M_{0.25\,E}}\right) < 1$ theoretically applies, not only to arches carrying no moment, but also to all arches symmetrically loaded; the condition is actually unnecessary, either because of initial inaccuracies or because of unsymmetrical loading (only canal bridges would not give unsymmetrical loading).

It is possible to apply the foregoing methods of calculation to the case of a hingeless arch. Considering the half arch as a uniformly loaded beam, hinged at the crown and fixed at the skewback, the moment (with the origin at the crown) is:

$$M = -\,w_a\left[\,(1 - \cos s{:}j)\,j^2 - \frac{\sin s{:}j}{\sin p}\,j^2\,J\,\right] \quad \ldots\ldots\ldots\ldots(75)$$

and the maximum occurs for $s = s_o$ such as

$$\tan\frac{s_o}{j} = \frac{J}{\sin p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(76)$$

in which $j^2 = -\,\frac{E\,I}{T}$; $p = \frac{L_c}{j}$; and L_c is the half length of the arch or a cor-

rected length taking into account the variations of moment of inertia. The load w_a is the antisymmetrical part, equal to one half of a live load distributed on half the span. The quantity J is equal to:

$$J = (1 - \cos p) - p^2 \frac{\tan \frac{p}{2} - \frac{p}{2}}{1 - p \tan p} \quad \ldots\ldots\ldots\ldots\ldots\ldots (77)$$

The maximum moment has the value

$$M = w_a j^2 \left(1 - \frac{1}{\cos s_o : j} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (78)$$

An approximate value of the maximum moment is (T_c = buckling thrust of a hinged arch):

$$M = \frac{9}{128} w_a (L_c)^2 \frac{1}{1 - 0.5\, T : T_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

When the arch is reinforced toward the skewbacks, the corrected length L_c is shorter than the actual development of the half arch and can be determined approximately by considering the angular deflection near the skewbacks. The equivalent length giving the same rotation as the actual length is $L_c = \int \frac{I_o}{I} ds$ in which I_o is for the basic section. For the Niagara Bridge it is approximately 470 ft. Thus the critical thrust is $T_c = -\frac{88.2 \times 29{,}500 \times 144 \times \pi^2}{470^2}$ $= -16{,}700$. With $T = -6{,}460$, the value c is 1.24. The total moment for $\overline{DL} + \overline{LL}$ is then $11{,}948 \times 1.24 + 2{,}138 = 16{,}950$, which differs by 2% from the value 16,628 calculated in Table 3. The error in the final stress due to the simplified calculation would be less than 1%. Thus, very simple calculations give accurate values of moments. If a higher degree of accuracy is sought, then the expressions developed in the Symposium are no more adequate than the foregoing simplified expressions.

Conclusions from the First Symposium Paper.—Conclusions 1 and 2 are incorrectly formulated, since the factor to be considered is the dimensionless quantity $\frac{T}{T_c}$ $\left(\text{or } \frac{T\,u}{M} \right)$ and not the factor $T\,u$. This factor has been well known for many years. Conclusion 2, as applied to heavy dead loads, is often incorrect. Let s_a = stress due to axial load, s_c = stress due to the axial load that produces a given buckling, and s_b = the stress due to bending, without amplification. It can be shown that for $\frac{s_a}{s_c} + \frac{s_b}{s_a} < 1$, an increase of dead weight effects a decrease of the influence of the coefficient of amplification on the total stress. Moreover, an increase of dead weight generally means an increase of the depth of the arch; thus, s_c is increased and the coefficient of amplification is reduced, which adds to the effect indicated. Considering the two arches analyzed from that point of view, an increase of dead weight on the hinged arch may cause a small increase of the flexibility effect on the total stress (increase limited to 2% for an increase of 40% of dead weight), whereas the fixed arch is

in the zone in which an increase of dead weight causes a decrease of the flexibility effect.

Much simpler design formulas than those indicated are available for many conditions of beam-columns and such formulas can be applied to arches without any difficulty.

Moments in Flexible Two-Hinged Arches.—The methods of analyzing problems of stress distribution by tests are generally more costly and require more time than mathematical approaches to the same problems. In the case of flexible two-hinged arches, the mathematical treatment is so simple and so commonly known that tests do not add any real knowledge and, if not interpreted with the use of the theory, may even be misleading. The interpretation of the tests made on two-hinged arches is an example of such a case.

It is here assumed that the moments due to dead weight have been eliminated by a change of span compensating for the shortening of the arch. Only in that manner can a pressure line coincide with the neutral line of the arch; but it is to be noted that the distribution of dead load is then far from an actual distribution, since the load must be inversely proportional to $\cos^3 \alpha$. For the three arches examined, this signifies that the ratio of dead load at skewbacks and at crown is equal, respectively, to 1.8, 2.6, 7.7. As it is improbable that actual variations of dead load will be greater than 40% between crown and skewbacks, the tests do not seem to represent actual conditions. On the other hand, in order to compare more accurately the results of tests and the theoretical results indicated hereafter, it would be desirable to know the exact law of variation which was used for dead load.

The moment at any point of an arch is equal to the moment of a nonflexible arch multiplied by the coefficient of amplification; using Eq. 14*a*, $M_{af} = \dfrac{M_{ae}}{1 - T_a u_{ae} : M_{ae}}$. The ratio, $\dfrac{M_{ae}}{u_{ae}}$, is a constant, equal to the critical thrust T_c, and T_a can be replaced by $R_{td}\,\overline{DL} + 0.5\,R_{tl}\,\overline{LL}$. That expression is exact (changes of length omitted) for a uniform load extending over one half of the span. For other distributions of live load, the factor 0.5 would be changed. It can be shown that (approximately) the ratio of thrust to live load R_{tl} is equal to $R_{td}\left[1 - \dfrac{2\,(\alpha_s)^2}{7}\right]$. Calling $\overline{DL}_o$ the dead load corresponding to the critical thrust T_c, and introducing a coefficient of moment m (ratio of moment to live load) such that $M_{ae} = m\,\overline{LL}$, the value of M_{af} is:

$$M = \frac{m\,\overline{LL}}{1 - \dfrac{\overline{DL} - \overline{LL}\,[0.5 - (\alpha_s)^2 : 7]}{\overline{DL}_o}} \qquad \ldots\ldots\ldots\ldots\ldots (80)$$

replacing $\overline{LL}$ in terms of $\overline{DL}$ and $R\left(= \dfrac{\overline{DL}}{\overline{LL}}\right)$:

$$M = \frac{m\,\overline{DL}}{R\left(1 - \dfrac{\overline{DL}}{\overline{DL}_o}\right) - \dfrac{\overline{DL}}{\overline{DL}_o}\left[0.5 - \dfrac{(\alpha_s)^2}{7}\right]} \qquad \ldots\ldots\ldots\ldots\ldots (81)$$

Thus, the curve representing M as a function of R is a hyperbola. Eq. 81 contains two constants m and $\overline{DL}_o$ instead of the four constants of Eq. 24, since the factor $\frac{(\alpha_s)^2}{7}$ generally may be neglected. Such constants can be computed

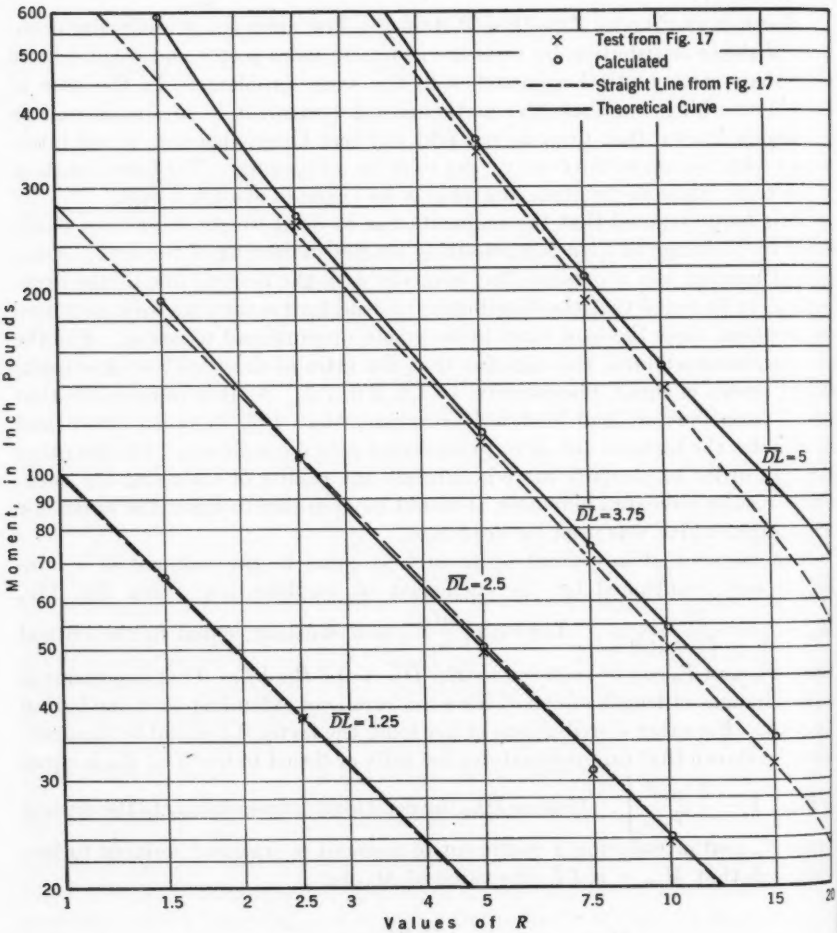

FIG. 41.—LOG-LOG PLOT OF MOMENTS

easily, and the final expression of the moment becomes

$$M = \frac{L^2\,\overline{LL}:64}{1 - \frac{(L')^2}{4\,\pi^2\,(r_o)^2} - \frac{(L')^2\,r_o}{4\,\pi^2\,E\,I}\left[1 + \frac{(\alpha_s)^2}{6}\right]\left\{\overline{DL} + \overline{LL}\left[0.5 - \frac{(\alpha_s)^2}{7}\right]\right\}} \quad \ldots (82)$$

The quantity $1 + \frac{(\alpha_s)^2}{6}$ takes into account the average value of the thrust in the

arch. Since $\frac{L'}{2\,r_o} = \alpha_s$, Eq. 82 can be written:

$$M = \frac{L^2\,\overline{LL}\,[1 + 0.1\,(\alpha_s)^2]:64}{1 - [1 + 0.6\,(\alpha_s)^2]\left\{\overline{DL} - \overline{LL}\left[0.5 - \frac{(\alpha_s)^2}{7}\right]\right\} L^2\,r_o:(4\,\pi^2\,E\,I)} \quad \ldots (83)$$

Fig. 41 affords a comparison between the calculated values and those read from Fig. 17 of the moments. The relative average difference is + 4.5% between the theoretical values and the tests. The average difference would be less if the dead load varied less than indicated by the law $\frac{1}{\cos^3 \alpha}$, and would change its sign for a relative variation equal to half of that given by such law.

As the relation between M and R is represented by a hyperbola, on a log-log diagram, the hyperbola becomes a curve with two asymptotic directions, one parallel to the M-axis and corresponding to buckling, and the other at an angle − 45° with the axes. The straight lines in Fig. 17 are the parts of the transformed hyperbolas which are far from the region of maximum curvature. Thus, they have no significance and do not even represent the asymptotic direction at 45°, since they show the beginning of the amplification effects. On the other hand, moments must become infinite when the loads approach buckling values. For instance, for $\overline{DL} = 5$ and 3.75, buckling occurs when $R = 1.5$ and 0.6, respectively. Therefore, it seems that Eq. 24 has no practical value, since its validity is limited to the range of the tests while Eq. 83 is valid for any values; and moreover it is more complicated and requires more computation than the theoretical expression, Eq. 80. It is to be noted that, in Eq. 24, the loads are in kips per foot and moments are in thousands of foot-kips. The authors have realized the advantage of the coefficient of amplification (called by them "percentage increase") but have made no attempt to introduce it on a rational basis and relate it to the critical thrust. With the use of the theory it is much easier for the designer to examine the influence of changes in shape of the arch and distribution of the load, and tests are unnecessary. Also, the main factors, span L, rigidity $E\,I$, dead load $\overline{DL}$, live load $\overline{LL}$ enter into a theoretical formula in a much more expressive manner than in a formula of the type of Eq. 24, which gives no measure of the relative influence of the various parameters.

The theoretical formula, Eq. 83, reduced to

$$M = \frac{L^2\,\overline{LL}}{64 - (2\,\overline{DL} - \overline{LL})\,L^4:(\pi^2\,h\,E\,I)} \quad \ldots\ldots\ldots\ldots (84)$$

will enable the designer to visualize the influence of the main factors, especially the rise of the arch.

Conclusions, Second Symposium Paper.—The conclusions advanced in this second paper of the Symposium do not throw any new light on well-known results. Tests of the kind made (conclusions 1, 2, and 3) are of little interest, since simple and reliable theories are available, and from such theories definite arches can be selected directly, either for bridges or for hangar roofs. A hingeless arch generally will prove less costly than a two-hinged arch, but there is no special reason to avoid the latter in cases where the coefficients of amplification

are small (say, less than 2) when it is difficult to fix the ends. In airplane design, for instance, high coefficients of amplification are accepted.

It is obvious that methods which do not take into consideration beam-column effects are unreliable (conclusion 4), but it is to be hoped that no designer will calculate flexible structures without considering buckling and prebuckling effects—that is, the coefficients of amplication.

There is no doubt that correctly chosen average values of radii and moments of inertia (conclusion 5) will enable the designer to make use of formulas or test data related to circular arches; but no difficulty is encountered in making direct calculations for any type of arch.

It is well known that the relationship between load and stress in beam-columns is not linear (conclusion 6). This fact is taken into account in various specifications for margins of safety—for instance in the Army-Navy-Civil Specifications for Aeronautics (see ANC-5), and also in many recent foreign specifications applied to civil engineering (French Air Department, Air Bases Division, dated about 1934 or earlier).

Since arches buckle in two waves and a change of length introduces a moment corresponding to one wave, the two phenomena are independent, and the nondeflectional theory is valid for temperature stresses (conclusion 7). However, the change of thrust due to temperature will change the coefficient of amplification; and thus there is an influence of temperature on buckling and on live-load moments. That effect, which is small for a bridge, is applied in automatic devices used in temperature control; and it cannot be disregarded entirely.

Nothing fundamentally new has been advanced in the analysis of the Rainbow Arch Bridge (conclusion 8).

It would be very desirable to know precisely what are the "inconsistencies with the elastic theory" (conclusion 9) which occur in the fixed-end type of arch.

Fabrication and Erection Controls.—Unusual care has been taken in the fabrication and erection of the Rainbow Arch Bridge (second Symposium paper), and the engineers who directed that work can consider it as a genuine achievement. It also appears that their computations have been made with all desirable accuracy.

It seems useful to note (see heading, "Application of Data to Niagara Rainbow Arch Bridge") that the elastic centroid concept is not due to Müller-Breslau,[17] since it was taught by Maurice Lévy[46] in France as early as the end of the last century in École Centrale des Arts et Manufactures. The correct expression, as used in treatises on mathematics, is "orthogonalisation" and not segregation; and the elastic centroid method is only a special case of the methods of "orthogonalisation" of the unknowns in systems of linear equations.

Moments can be applied at the crown instead of the centroid, since the axis of a moment can be freely displaced without changing the equilibrium.

The elastic center can be determined by a straightforward method (column analogy), which is still valid when deflections due to normal and shearing forces are taken into account.

[17] "Die graphische Statik der Baukonstruktionen," by H. Müller-Breslau, Vol. 2, Pt. 2, 1925, p. 377 *ff.*

[46] "La Statique Graphique et ses Applications aux Constructions, 1886–1888," Gauthier-Villars & Co., Paris.

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DISCUSSIONS

STORAGE AND THE UNIT HYDROGRAPH

Discussion

BY L. K. SHERMAN, GORDON R. WILLIAMS, AND RAY K. LINSLEY, JR.

L. K. SHERMAN,[13] M. AM. SOC. C. E.[13a]—The definite, but complex, relations that exist between the hydrograph of surface runoff, the procedure of flood routing, and the law of proportionality of certain hydrographs, are presented in this paper.

All the objectives noted in the "Synopsis" have been fulfilled, with one very important exception. That exception is the author's statement, "to derive accurate unit hydrographs." No unit hydrographs or percentage distribution graphs have been derived in this paper.

Col. 10, Table 1, is labeled "12-hr unit hydrograph (cu ft per sec)." This caption is very misleading to a reader. It is not a unit hydrograph. The definition of a unit hydrograph, accepted and used since 1932, is: The hydrograph of surface runoff from a given basin due to a 1-in. depth of net rainfall, applied at a uniform rate in a unit of time. The ordinates of flow of this unit hydrograph (expressed in cubic feet per second) and the ordinates of flow of an observed hydrograph, due to a net rain of X inches in the same unit time, are proportional to their respective volumes of total surface runoff.

Net rain means rainfall minus infiltration and other losses.

The distribution graph is the unit hydrograph with ordinates expressed in percentage of the total volume of the observed hydrograph. The percentage distribution graph is commonly used in practice.

The proportionality of ordinates holds true for any two hydrographs of surface runoff on the same basin, provided the net rains in each case hold a direct relation of intensities, and follows identical sequence of downpours.

Let it be required to apply the foregoing criteria to this Appomattox River basin with a drainage area of 1,335 sq miles. If 1 in. of net rain fell on this basin, and if the rain followed identical times and similar intensities as that of April, 1937 (see Fig. 9), it would produce a hydrograph with the same time base

NOTE.—This paper by C. O. Clark was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by James S. Sweet, and Otto H. Meyer.

[13] Cons. Engr., Chicago, Ill.

[13a] Received by the Secretary February 7, 1944.

as that observed in Fig. 9. The ordinates of runoff would be proportionate to the ordinates in Fig. 9, and the volume of surface runoff would be 1,335 in.-miles or 36,000 (cu ft per sec)-days or 72,000 (cu ft per sec)-half-days. The total surface runoff (by scaling ordinates in Fig. 9) in the flood of April, 1937, is 215,000 (cu ft per sec)-half-days. The base flow (also estimated from Fig. 9) was, approximately, $\frac{1,000 + 2,000}{2} \times 19$ half-days or 29,000 (cu ft per sec)-half-days. Then 216,000 − 29,000 = 187,000 (cu ft per sec)-half-days as the total volume of surface runoff due to the April, 1937, storm.

A similar storm producing 1 in. of runoff from this basin was 72,000 (cu ft per sec)-half-days.

The average depth of net rain, measured from the surface runoff hydrograph, is 187/72 = 2.6 in. of average depth over the Appomattox basin in the flood of April, 1937.

The comparable figures given in Table 1 are: Runoff for 1 in. of net rain = 60,000 (cu ft per sec)-half-days. This is the total of Col. 10. This amount is 87% of the total runoff. Therefore, 100% of 1 in. of net rain is 69,000 (cu ft per sec)-half-days. Accordingly, the depth of net rain utilizing the author's figure is 187/69 = 2.7 in.

This agreement by the author's elaborate routing analysis is an ingenious and possibly very useful accomplishment. It has no value as a derivative of the ratio of 1 in. of net rain to the total runoff from a particular storm. That is found readily in a few minutes from an observed hydrograph and a known size of drainage area. Col. 10, Table 1, is the runoff of 1 in. of net rain over the basin; but it certainly is not 1 in. applied over the basin in one unit of time. Therefore, it is not a unit hydrograph or a distribution graph. It certainly cannot be used to derive the hydrograph of runoff from the Appomattox basin under a storm of entirely different pattern. That is what the unit hydrograph is for. The same criticism applies to all the data in Table 2.

The author states (see heading, "Derivation of the Instantaneous and Unit Hydrographs") that another storm "is equally well reproduced with the same [so-called] unitgraph." He has not demonstrated the truth of this statement.

However, it would seem possible to utilize the methodology of Table 1. In this case the data furnished no record of the amounts of unit-time rainfalls on the basin. The total net rainfall averaged over the basin is simply and readily found as 2.6 in. or 2.7 in. How much of this fell in each of how many half-day periods is unknown. If this were furnished or found, a unit hydrograph could be derived. The unit hydrograph could be derived just as well if the 12-hr rainfalls for the period, April 25–29, were known—by the methods described by Mr. McCarthy[9] or the writer.[14] In deriving a unit hydrograph, the first step is to find the volume or volumes of net rainfall that fell in unit periods of time. These are the volumes that produce the hydrograph of surface runoff.

[9] "The Unit Hydrograph and Flood Routing," by Gerald T. McCarthy, unpublished manuscript presented at the Conference of the North Atlantic Div., Corps of Engrs., U. S. Army, New London, Conn., June 24, 1938; printed by the U. S. Engr. Office, Providence, R. I.

[14] "Physics of the Earth," National Research Council, Div. of Physical Sciences, Pt. 9—"Hydrology," by O. E. Meinzer, 1942, p. 521.

In the author's example of April, 1937, in Fig. 9, the total runoff was found to be 2.7 in., which is the sum of several unit-time downpours; but, in this case, the number or relative amounts of the unit volumes which total 2.7 in. is not known.

The bar pattern, Fig. 8, curve (a), indicates that the author has discovered that the 2.7-in. runoff was due to 12 unit periods of rain with ordinates proportional, or equal, to this bar pattern. Mr. Clark does not describe how he found this bar pattern. The number of blocks agrees with the number of time units in Table 1, Col. 2. The ordinates or heights of these blocks agree fairly well with Col. 2 averages.

The total percentage of Col. 2, however, is not 100%, or even 87%, to correspond with the total of Col. 8, Table 1.

It is hoped that, in his closure, the author will consider this feature of Table 1 and the block pattern (a) of Fig. 8. He may have found a way of determining the unit volumes of net rainfall which created the observed hydrograph, without prior data on rainfall. Astounding as this statement may seem, there is nothing fundamentally impossible about it. The hydrograph of surface runoff reflects, not only the physical characteristics of the basin, but also the pattern or sequence of the creating net rainfall. In other words, with an observed hydrograph of surface runoff, it is possible to derive the unknown unit rains and thus the unit hydrograph. With the unit hydrograph and the observed rainfall, the corresponding hydrograph of runoff is found readily.

If the author presents further consideration of this feature, it is also hoped that he will follow the orthodox procedure of eliminating base flow. The problem is sufficiently complex without including it. In this connection, attention is called to the work of Robert E. Horton, M. Am. Soc. C. E., which includes routing and the derivation of unit hydrographs.[15]

The writer does not agree with the author's statement that 1,000 sq miles is a large basin upon which to apply the unit hydrograph or distribution graph. The distribution graph has been frequently and satisfactorily used for basins of 5,000, 10,000, and even 30,000 sq miles. The theory of the unit hydrograph assumes a uniform rate of rain for a relatively small fraction of the time of runoff. The volume in this unit time is the same volume as the nonuniform rain in this unit time. Each unit volume changes with the storm intensity. If the unit times are taken small enough, the runoff hydrograph will be exactly reproduced.

In practice, whether to use small areas and apply routing, or use large areas without routing, depends altogether upon the available data including the number of stream gages in the basin.

GORDON R. WILLIAMS,[16] ASSOC. M. AM. SOC. C. E.[16a]—Numerous positive statements are made by the author which controvert accepted concepts and experience in the field of river hydraulics. These statements are not sub-

[15] "Virtual Channel-Inflow Graphs," by Robert E. Horton, *Transactions*, Section of Hydrology, Am. Geophysical Union, Pt. III, 1941, p. 818 (4).

[16] Senior Engr., Office of the Chf. of Engrs., U. S. Army, Washington, D. C.

[16a] Received by the Secretary February 10, 1944.

stantiated in the paper by experimental evidence or observations of actual conditions.

The most radical theory deals with "pressure" or momentum waves moving downstream through reservoirs and natural river reaches. The author claims that, if the inflow to a reservoir or river reach is decreased abruptly, the slope storage in the upper end of the reservoir or reach moves downstream as a pressure wave and increases the stage and discharge at the lower end of the pool or reach. Laboratory experiments have been conducted in which "decrement waves" were produced in artificial channels.[17] Such waves move downstream in accordance with the equation,

$$u = v_1 + \sqrt{g\,d_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which u = wave velocity; g = acceleration due to gravity; d_1 = initial depth; and v_1 = initial velocity of flow. It is doubtful whether the natural inflow to a reservoir or river reach could decrease fast enough to create a true momentum wave. The contention that such a wave would cause an increase in stage and discharge at a downstream point was not substantiated by Robert E. Horton, M. Am. Soc. C. E., in his experiments. The author refers to an incident in which the effect of a levee break was transmitted downstream as a decrement wave at the rate of a mile a minute. The river would have had to be about 240 ft deep to transmit a momentum wave at the rate of 88 ft per sec.

A decrease in inflow to a river reach might be accompanied by an increase in elevation, but not in discharge, at the lower end of a reach in a river having an erodible bottom or in a river carrying a large bed load. In such a case the decrease in inflow would cause a flattening of slope and a loss of erosive power and carrying capacity, which in turn would result in a filling in of the channel and a possible increase in elevation. Such a phenomenon is basically different from the author's theory, but would give evidence that might lead to such a theory.

The statement is made (see heading, "Valley Storage") that the aforementioned phenomenon "takes place in a natural channel * * * by means of pressure waves of small amplitude which surge back and forth through the reach * * * ," and that "the surge * * * is clearly apparent as a broad trace in many autographic records of stage." This surge, which is evident on gaging station charts, is very common, particularly at high stages, and is merely a local disturbance at the station, due to turbulence in the stream. The surge is often most pronounced on swift, turbulent streams where the presence of shooting flow and even small waterfalls eliminates any possibility of pressure waves traveling up and down the stream. Surges in gaging station wells can often be reduced by modifications in the size and type of intake connection.

RAY K. LINSLEY, JR.,[18] JUN. AM. SOC. C. E.[18a]—The author's discussion of storage, the unitgraph, and accuracy in engineering is a valuable contribution to hydrology.

[17] "Channel Waves Subject Chiefly to Momentum Control," by Robert E. Horton, *Bulletin TP-16*, SCS, U.S.D.A., May, 1938.

[18] Associate Hydrologic Engr., U. S. Weather Bureau, Sacramento, Calif.

[18a] Received by the Secretary February 14, 1944.

The concept of storage which may act to increase outflow is familiar to all persons who have worked with reservoirs storing appreciable volumes of water under the backwater profile. The navigation pools of the Upper Mississippi and Ohio rivers are examples of this effect. To the writer's knowledge the author is the first to apply this concept to storage in natural streams. Obviously, under the proper conditions it can apply, but the discussion should be approached with some caution.

In Fig. 1 it will be noted that, as the water-surface profile changes from b to d, the slope of the water surface at 0 has been decreased. Since values of velocity and hence $v^2/2\,g$ upstream from 0 have been lowered, the slope of the energy gradient is decreased even more than that of the water surface. Using the data of profiles a and b the writer computes $1.486/n$ in Manning's formula to be 54.3. Assuming s equal to the slope of the water surface from station 2 to 0, the discharge at 0 under profile d becomes 108,000 cu ft per sec, whereas for profile f it would be 130,000 cu ft per sec. By approximation the slope of the energy gradient at station 0 for profile f seems to be 0.00055 ft per ft which gives a discharge of only 108,000 cu ft per sec.

Although these discharge values are only approximate, they serve to demonstrate the close interrelation between stage, discharge, and storage which must be considered in any discussion of river hydraulics. Actually, in the example of Fig. 1, it appears that the change from profile b to f would absorb the storage released by the decreased inflow through a rise in stage but with a decrease in outflow.

The writer has never observed a natural stream in which the release of inflow-controlled storage acted to increase outflow. On numerous occasions the writer has observed the effect of levee breaks or the operation of diversion weirs. The usual reaction is a sudden sharp fall in stage at the downstream station which is transmitted, as the author indicates, with great rapidity. Generally, however, the fall in stage is small in comparison to the volume of water diverted. This is explained by the two factors—(1) loss in discharge capacity at the station by reason of the reduced slope and (2) the release of a volume of inflow-controlled storage. However, since a considerable portion of this inflow-controlled storage is withdrawn by the diversion, the effect of the lost discharge capacity seems to be the important factor.

In natural channels the rates of change of flow, especially of decrease of flow, are usually too slow to permit a sufficiently rapid release of inflow-controlled storage to produce an increased downstream discharge. Possibly the abrupt cessation of rainfall from an intense local thunderstorm over a small basin or sharp decrease in discharge from a reservoir might accomplish such a result.

It is possible that, for some reservoirs, the decrease in discharge capacity of the outflow weir, as a result of a decrease in velocity of approach, may account for all or a portion of the rise in stage at the dam when the inflow is reduced.

The general scheme of deriving a unit hydrograph by adjusting a time-area concentration curve for valley storage has been discussed by several writers. The method outlined by the author should permit the determination of unit

hydrographs with a minimum of basic data, a minimum of disagreement between separate investigators, and a high degree of relative accuracy. It represents a considerable advance over earlier procedures.

The author makes the point that the ordinate of a unit hydrograph for any unit period will be equal to the average ordinate of the instantaneous hydrograph for the previous unit period. This is demonstrated easily since the ordinates of the time-flow concentration curve of Fig. 4(*b*) bear the same relationship to the time-area concentration curve of Fig. 4(*a*). Inasmuch as the routing method assumes storage to be directly proportional to flow, the relationship is not disturbed by any manipulation which follows. The relationship is not peculiar to instantaneous hydrographs, however. If two 6-hr unitgraphs are added with a 6-hr lag and the sum is divided by two, a 12-hr unitgraph results. The ordinate of the 12-hr graph at hour 12:00 is the sum of the 6-hr and 12-hr ordinates of the 6-hr graph divided by two, or the average ordinate of the 6-hr graph during the first twelve hours.

The author indicates an attempt to correlate storage with slope in order to devise a method for determining K for basins without adequate data. He found that the relation could be expressed approximately as: $\frac{K}{L} = \frac{1}{\sqrt{s}}$; or,

$$C_1 = \frac{K\sqrt{s}}{L} \qquad (18a)$$

in which L is the length of the stream in miles. Eq. 18*a* considers only the distance along the main stream and the slope. It would seem that other factors of importance would be the storage in tributaries and the comparative magnitude of flood flows to which the streams are subjected. If drainage area were introduced into Eq. 18*a*, it should indicate the effect of these two factors. Then: $\frac{K}{L} = \frac{\sqrt{A}}{\sqrt{s}}$; or,

$$C_2 = \frac{K\sqrt{s}}{L\sqrt{A}} \qquad (18b)$$

The writer has used the square root of A because peak flows tend to vary as the square root of the drainage area, and, in all probability, tributary storage also varies as some power of drainage area less than one.

Table 3 affords a comparison between Eqs. 18*a* and 18*b*. The first four items are from data given by the author, and the last seven are based on data available to the writer. Unfortunately, determinations of K for streams of the Sierra Nevada drainage are complicated by the presence of a snow pack which effectively reduces the drainage area, length of channel, and time of concentration, in addition to the other problems indicated by the author. The criterion of Eq. 18*b* seems to be more nearly a constant than does the value of $K\sqrt{s}/L$ from Eq. 18*a*. Values of $K\sqrt{s}/L\sqrt{A}$ have somewhat more of tendency to be higher for the California streams than do the values of Eq. 18*a*. This may indicate that a variation must be expected from region to region, but more likely it indicates that some power other than the $\sqrt{s}$ should be used.

Inspection of Fig. 6 indicates that the author divided the drainage area on the basis of equal units of stream mileage. Either Eq. 18*a* or Eq. 18*b* could be used to determine the subareas and should give a time-area concentration graph which more closely approximates the true graph. Eq. 18*a* is the

TABLE 3.—COMPARISON BETWEEN EQS. 18*a* AND 18*b*

Stream	s (ft per mile)	L (miles)	K (hours)	A (sq miles)	C_1 (Eq. 18*a*)	C_2 (Eq. 18*b*)
Appomattox River near Petersburg, Va.	2.9	122	144	1,335	2.01	0.055
James River at Lick Run, Va.	17.3	78	28	1,369	1.49	0.040
Smith River at Bassetts, Va.	16.6	37	7	265	0.77	0.047
Meherrin River at Emporia, Va.	2.4	86	78	750	1.40	0.051
Middle Fork, Feather River at Bidwell Bar, Calif.	59	98	16	1,353	1.26	0.046
American River:						
North Fork, at Colfax, Calif.	80	50	8	308	1.43	0.082
North Fork, at Auburn, Calif.	80	56	10	619	1.60	0.064
South Fork, at Coloma, Calif.	91	60	12	635	1.90	0.075
Kings River:						
At Piedra, Calif.	72	86	9	1,694	0.89	0.036
Above North Fork, Calif.	132	60	7	956	1.34	0.043
San Joaquin River above Big Creek, Calif.	192	40	7	1,042	2.22	0.075

simplest to apply, and, since the problem is merely to divide a basin for which K is already known, should be nearly as satisfactory as Eq. 18*b*. The number of reaches will equal K/T, and the values of $d/\sqrt{s}$ (d is the length of the reach) for all reaches should be equal. The solution is reached easily by approximations if a family of curves showing the variation of $d/\sqrt{s}$ with d, and with s, is plotted. A first approximation to the proper value of $d/\sqrt{s}$ will be the value of $T\,L/K\sqrt{s}$, which, for the Appomattox River, is 6.0. A final value of 5.75 was determined by the writer, which gives ten reaches of 8.6 miles, one reach of 12 miles, and one of 24 miles.

Corrections for *Transactions:* In November, 1943, *Proceedings*, page 1352, line 15, change "the time-area concentration curve;" to read "the time-area concentration curve converted to flow for 1 in. of runoff;" and, in Fig. 8, change the ordinate caption to read "Discharge in Thousands of Cu Ft per Sec, per 1-In. Runoff."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

Discussion

BY R. K. BERNHARD

R. K. BERNHARD,[24] M. AM. SOC. C. E.[24a]—In the author's unusually interesting paper the derivation of simple working formulas for the practicing engineer to predetermine the aerodynamic stability of suspension bridges represents, as far as the writer is aware, the first direct attempt to solve this difficult problem. The author has treated the entire range of this subject in a most comprehensive way. His primary aim—formulating positive rules to prevent failures due to aerodynamic instability or, as he expresses it, the prevention of this disease rather than its cure—seems of utmost importance.

In connection with the vibration of bridges, the logarithmic decrement for structural damping is of particular interest. The author presents a wide range of damping values for suspension bridges of the order of magnitude from 0.002 to 0.130. These damping decrements may be compared with damping values as cited in other publications.

In the Society's report on the Tacoma Narrows Bridge failure values from 0.0055 to 0.057 are published resulting from tests[25] by F. B. Farquharson, Assoc. M. Am. Soc. C. E., on a suspension bridge model.

Experiments, started as early as 1928 by the writer on actual bridges with the method of induced vibrations, yielded values for structural damping from 0.063 to 0.112.[26,27] The lower values refer to welded beams, the higher values to riveted trusses, and the values in between to riveted through girders. It seems obvious that higher stresses in the connections of riveted trusses cause some relative motion and hence energy losses, thus resulting in larger damping.

NOTE.—This paper by D. B. Steinman was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by Edward Adams Richardson.

[24] Trenton, N. J.

[24a] Received by the Secretary February 3, 1944.

[25] *Proceedings*, Am. Soc. C. E., December, 1943, p. 1580.

[26] "Purely Dynamic Methods of Investigating Stresses in Structures," by R. Bernhard and W. Späth, *Der Stahlbau*, March 22, 1929, p. 61; also "Endurance Tests on Riveted and Welded Bridges," by R. K. Bernhard, *Verein deutscher Ingenieure Zeitschrift*, Vol. 73, No. 47, 1929, p. 1675.

[27] "Determination of Static and Dynamic Constants by Means of Response Curves," by R. K. Bernhard, *Journal of Applied Physics*, Vol. 12, No. 12, December, 1941, pp. 866–874.

For similar reasons an increase in damping with rising amplitudes could be experienced coinciding approximately with Eq. 35 which contains a factor $\sqrt{a_m}$ (a_m = amplitude). Furthermore a certain change of the damping capacity with respect to the number of vibrations induced in the structure or structural unit could be observed. During a series of long-time vibration tests at first a decrease and, at impending failure, an increase, in structural damping was noticeable. The same method of induced vibrations yielded somewhat similar results for the internal damping capacity in small mild steel specimens.[28]

In general, damping values determined by experiments on actual bridges of any type may indicate higher figures, not only when compared with metals but also with bridge models. The previously mentioned damping effect of structural connections, the damping of the foundations, and, finally, the damping of the surrounding soil, etc., will be noticeable, all these effects being rather difficult to imitate in model tests. Referring to the influence of soil characteristics, average damping values as high as 1.25 have been observed in actual field tests.[29]

In the Society's report of the Tacoma Narrows Bridge failure[25] (Appendix IV) an "accepted value of 0.3 for beam type bridges" is cited. According to this figure the logarithmic damping decrement for beam-type bridges is approximately 2.3 to 150 times higher than that for suspension bridges mentioned by the author. The value of 0.30 approaches rather closely the value of 0.31 as published in 1937.[29] It should be kept in mind that this high value, however, represented an average value for all types of structural units such as engine parts, ceilings, walls, towers, ship decks, masts, airplanes, and airship parts.

The author rightly emphasizes the fact that accurate information on the structural damping of suspension bridges is lacking. Furthermore a reduction of the wide range of damping values as published by the various investigators might be valuable. Both seem to indicate that experiments on actual suspension bridges are desirable.

In his conclusions the author states that a modification of the bridge cross section, by introducing open spaces in the floor, can be used to eliminate aerodynamic instability at the source.

This fact has also been observed by the writer on a simple model used for classroom demonstration at Pennsylvania State College, at State College, in 1940 and 1941. The model consisted essentially of a small wooden frame of approximately 10 in. by 10 in. suspended by four weak vertical springs, one in each corner. The conditions for a vibrating system with six degrees of freedom were fulfilled; in other words, oscillations of the sides or of the corners of the frame either in phase or in antiphase were possible. Since the wooden frame did not deform during the vibration, only the almost negligible internal damping of the four springs remained active. The front and rear bar of the wooden frame could be replaced by rods of aerodynamically stable or unstable cross section. A removable piece of cardboard, stretched horizontally between the front and rear bar of the frame, imitated, at least to a certain extent, the deck

[28] "Some Mechanical Properties of Plastics and Metals under Sustained Vibrations," by B. J. Lazan, *Transactions*, A.S.M.E., February, 1943, p. 87.

[29] "Dynamic Tests by Means of Induced Vibrations," by R. K. Bernhard, *Proceedings*, A.S.T.M., Pt. II, 1937, pp. 634–645.

of one bridge panel. Thus an atmospheric damping factor, as mentioned by the author, was introduced. A constant horizontal wind stream, produced by an ordinary fan, was directed against the model. Different types of linear as well as torsional self-sustained vibrations could be built up. These experiments can be considered as elementary pilot tests only.

A rectangular section of the front and rear bar of approximately 2 in. by 1 in., the larger side facing the wind stream, simulated unstable conditions for the individual girder. Whenever the cardboard deck remained in place the amplitudes increased up to a definite dynamically stable equilibrium. Removing this cardboard completely resulted in increased amplitudes, probably caused, in part, by the substantial reduction in atmospheric damping.

Another set of bars with half-moon cross sections of approximately 1-in. radius, the flat sides facing the inside of the frame, simulated dynamically stable conditions for the girder nearest the fan. No substantial reduction of the maximum amplitudes (that is, shielding effect) could be observed since the rear girder remained unstable.

Finally a change in cross section of the complete bridge panel, for example by cutting longitudinal holes in the cardboard deck, corresponding approximately to the width of the sidewalks, reduced rather effectively the excitation of any type of self-sustained vibration regardless of the cross section that was substituted for the individual girders.

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DISCUSSIONS

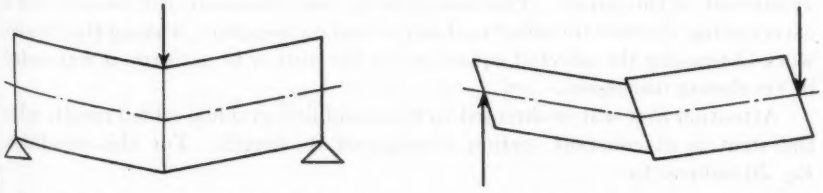
SHEAR EFFECT ON THE STRENGTH OF STRUTS

Discussion

BY DAVID B. HALL

DAVID B. HALL,[16] Assoc. M. Am. Soc. C. E.[16a]—This paper, together with another by the same author,[17] which it supplements, is a useful addition to the literature of column analysis. Hence, with some regret the writer notes an error in the derivation which will necessitate some revisions.

The error is in Eq. 21, which was introduced to establish continuity between the two segments of the strut shown in Fig. 1. The author has assumed that the elastic curves of both segments must have the same slope at the point of application of the concentrated load P, where they join. This assumption is incorrect. The slope of the elastic curve of shearing deflection of a beam is



(*a*) EFFECT OF CONCENTRATED LOAD (*b*) EFFECT OF CHANGE OF SECTION

FIG. 7.—SHEARING DEFLECTION CURVES

discontinuous at points of concentrated loading (Fig. 7(*a*)), and at points of abrupt change in section (Fig. 7(*b*)).

The correct method of establishing compatibility at $x_a = a$ is to fit together the two faces of the section cut through that point. The expressions for the slopes of cross sections of the upper and lower segments of the strut are shown in Fig. 8. This figure is diagrammatic. In solid members, for example, plane

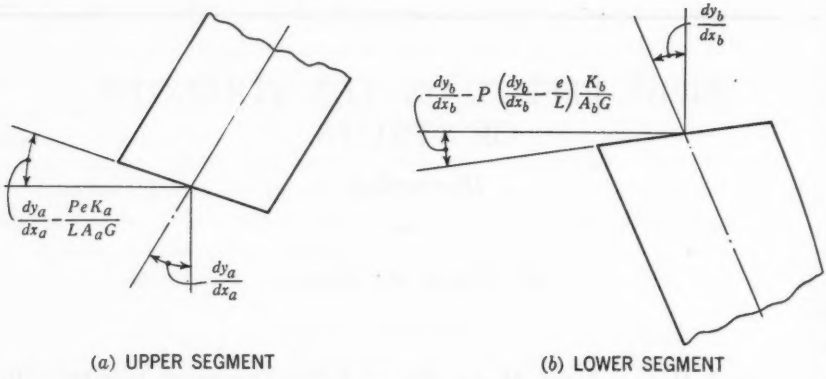
NOTE.—This paper by S. Sergey was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by Leon Beskin, and Marshall Holt.

[16] Stress Analyst, Otis Elevator Co., New York, N. Y.

[16a] Received by the Secretary February 3, 1944.

[17] *Bulletin No. 101*, Eng. Experiment Station, Univ. of Washington, Seattle.

sections under shearing loads become **S**-shaped. The sloping plane surfaces shown represent effective values, obtained with the aid of the shape factor K in the case of solid members, or by means of Eqs. 28 in the case of built-up members.



(*a*) UPPER SEGMENT (*b*) LOWER SEGMENT

FIG. 8.—RELATION OF SLOPE OF CROSS SECTION TO SLOPE OF ELASTIC CURVE

To make the lower face of the upper segment match the upper face of the lower segment one must write, in place of Eq. 21:

$$\left(\frac{dy_a}{dx_a} - \frac{P\,e\,K_a}{L\,A_a\,G}\right)_{x_a=a} = -\left[\frac{dy_b}{dx_b} - P\left(\frac{dy_b}{dx_b} - \frac{e}{L}\right)\frac{K_b}{A_b\,G}\right]_{x_b=b} \quad \ldots (56)$$

The substitution of Eq. 56 for Eq. 21 must have some effect on all further development in the paper. The remainder of this discussion will be confined to ascertaining whether the effect is of any actual consequence, leaving the detailed work of revising the affected equations for the author to perform, if warranted, in his closing discussion.

Attention first will be directed to the simplified problem which results when the strut is of constant section throughout its length. For this condition, Eq. 56 reduces to

$$\left(\frac{dy_a}{dx_a}\right)_{x_a=a} = -\frac{dy_b}{dx_b}\left(1 - \frac{P\,K}{A\,G}\right)_{x_b=b} \quad \ldots (57a)$$

or

$$\left(\frac{dy_a}{dx_a}\right)_{x_a=a} = -\frac{m^2}{q^2}\left(\frac{dy_b}{dx_b}\right)_{x_b=b} \quad \ldots (57b)$$

Also, $u = 1$. The operations which immediately follow are analogous to those of the paper, and, in place of Eq. 24*b*,

$$\frac{\tan k\,q\,L}{q\,L} = \frac{1}{\dfrac{q^2\,L^2}{3}(1-k) - \dfrac{\dfrac{q^2}{m^2} + 1 - k}{(1-k)^2}} \quad \ldots (58)$$

For purposes of comparison it will be convenient to convert both Eq. 24*b* and Eq. 58 into the terms introduced in Eq. 30, with some modification. In the paper t is a measure of the critical column load, and α is a measure of the relative shearing flexibility. Now designate all corresponding terms pertaining to a strut subject to bending deflection only by the subscript M. Let:

$$t = t_M + \delta t \quad \ldots\ldots (59)$$

Then

$$m_M L = t_M \quad \ldots\ldots (60a)$$

$$m L = t_M + \delta t \quad \ldots\ldots (60b)$$

and

$$q L = \frac{t_M + \delta t}{[1 - \alpha (t_M + \delta t)^2]^{0.5}} \quad \ldots\ldots (61a)$$

It will be of further advantage to confine the comparison to struts for which α is a small number. As α approaches zero, δt will also approach zero, and a linear relation between them can be found by the same processes used in deriving the rules for differentiation. Eq. 61*a*, for example, may now be rewritten:

$$q L = t_M + \delta t + \tfrac{1}{2} \alpha t^3_M \quad \ldots\ldots (61b)$$

By (1) making the indicated substitutions, (2) carrying out the process of differentiation, and (3) subtracting from each side of the equation the terms that would appear in the equation for a strut subject to bending deflection only—one will finally arrive at the following new equations: From Eq. 24*b*—

$$(\delta t + \tfrac{1}{2} \alpha t^3_M) \left(\frac{k \sec^2 k t_M}{t_M} - \frac{\tan k t_M}{t^2_M} \right) = \frac{- 2 t_M \delta t (1 - k)}{\left[\frac{t^2_M (1 - k)}{3} - \frac{2 - k}{(1 - k)^2} \right]^2} \quad \ldots\ldots (62a)$$

and, from Eq. 58—

$$(\delta t + \tfrac{1}{2} \alpha t^3_M) \left(\frac{k \sec^2 k t_M}{t_M} - \frac{\tan k t_M}{t^2_M} \right) = \frac{- 2 t_M \delta t (1 - k) - \alpha t^2_M \left[\frac{t^2_M (1 - k)}{3} - \frac{1}{(1 - k)^2} \right]}{\left[\frac{t^2_M (1 - k)}{3} - \frac{2 - k}{(1 - k)^2} \right]^2} \quad \ldots\ldots (62b)$$

Numerical values of t_M, or $k t_M$, as a function of k are already known,[6] and may be substituted in Eqs. 62*a* and 62*b*. Thus values of $\delta t / \alpha$ may be calculated as a function of k for each of these equations, Eq. 62*a* representing the author's derivation and Eq. 62*b* representing the corrected derivation. The two functions are plotted together in Fig. 9. The principal relations between these two curves can easily be interpreted: (*a*) The lateral component of P, producing the effect shown in Fig. 7(*a*), acts inward when P is applied in the upper part of the strut, and outward when P is applied in the lower part of the strut, thus

[6] *Bulletin No. 101*, Eng. Experiment Station, Univ. of Washington, Seattle, p. 18.

increasing the stability in the former case and decreasing the stability in the latter case; and (*b*) the part of the shearing effect considered by the author—namely, the change in curvature in the loaded segment—diminishes in importance as the load approaches the lower support; hence, the relative error is greatest for small values of k.

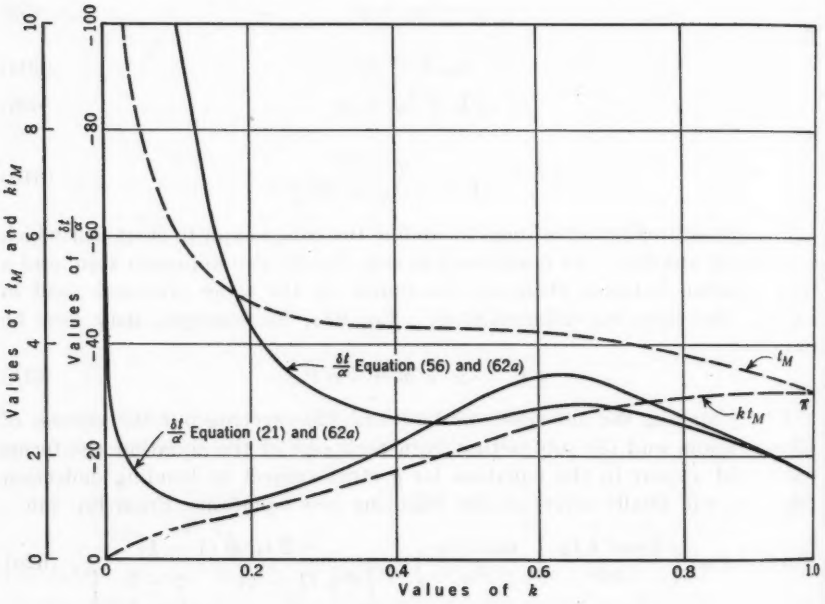


FIG. 9.—VALUES OF $\frac{\delta t}{\alpha}$ COMPUTED FROM EQS. 62*a* AND 62*b*

The problem is more complicated when there is a change of section as in Fig. 7(*b*). Under ordinary conditions of design, the loaded segment of such a strut as that shown in Fig. 1 will be made stronger than the unloaded segment, and in many cases will be more resistant to shearing deflection. Under this, the normal condition, the angle at the junction of the two segments, produced by the shearing force $P\,e$, will always be in such a direction as to increase the deflection, and hence decrease the stability of the strut. In a case where the upper segment of the strut is considerably more yielding than the lower segment, and the load is applied at approximately midheight, this phenomenon may prove a substantial, or even the predominant, part of the total shear effect. Where the load is applied near either end, this effect will be less.

Basing his judgment on the foregoing comparisons, the writer believes that the error in the paper is sufficiently large to warrant revision of the equations affected.

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DISCUSSIONS

SIMPLIFIED ANALYSIS OF SKEWED REINFORCED CONCRETE FRAMES AND ARCHES

Discussion

BY MAURICE BARRON, PHILLIPS H. LOVERING, AND JAROSLAV POLIVKA

MAURICE BARRON,[27] ESQ[27a].—In 1932 the writer, under the author's supervision, made some of the computations for an investigation of the effect of variation in the skew angle on the reactions of skewed frames. The method then used was that first published in 1931 by Mr. Hayden[28] and the investigation was carried through the simultaneous solution of the elastic equations. In this investigation the writer was concerned with a structure of the standard concrete rigid frame type of bridge. Only the skew angle was permitted to vary. The conclusion was that, for all practical design purposes, and well within the limits of computational accuracy, the rectangular horizontal reaction is independent of the skew angle. This conclusion is the basis for the entire first part of the paper dealing with analysis.

The writer has investigated the validity of this conclusion when applied to two reinforced concrete elliptical skewed barrel arches (for convenience identified as bridges A and B). Bridge A, designed by the writer, is a 45° skewed arch with a square span of 56 ft. Bridge B has a 36.5° skew and a square span of 57 ft.

In designing bridge A many short cuts (not approximations) were used including the principle of the elastic centroid, ordinary and torsional elastic weights, the shear increment method of computing moments, and Maxwell's theorem of reciprocal deflections. The four elastic equations (see Appendix, Eqs. 7) were thus greatly simplified; one or more coefficients in each equation, a total of eight, were reduced to zero. The solution of the four equations was correspondingly simplified and resulted in the following equation for the rec-

NOTE.—This paper by Richard M. Hodges was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. C. D. Geisler, and Arthur G. Hayden; November, 1943, by Bernard L. Weiner, and Alfred L. Parme; and February, 1944, by J. Charles Rathbun.

[27] Designer, Office of Gilmore D. Clarke, New York, N. Y.

[27a] Received by the Secretary February 7, 1944.

[28] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed.

tangular horizontal reaction: $65.7574\ R_x = +\delta_x + 1.042448\ \delta_z - 3.197289\ \phi_y$; or,

$$R_x = 0.0152074\ \delta_x + 0.0158529\ \delta_z - 0.0486225\ \phi_y \quad \ldots\ldots\ldots (26)$$

in which R_x is the rectangular horizontal reaction; δ_x is the deflection (including torsional effects) of the base frame in the direction and at the point of application of R_x, due to the load under consideration; δ_z is the deflection of the base frame in the direction and at the point of application of R_z, due to the load under consideration; and ϕ_y is the angular rotation of the base frame in the direction of M_y, due to the load under consideration.

For the proposed simplified method:

$$65.37016\ H = \delta'_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which H is the rectangular horizontal reaction; and δ'_x is the deflection of the rectangular structure in the direction and at the point of application of H, due to the load under consideration. The coefficient of H is $\Sigma \frac{y^2}{I} \Delta s$ and does not contain torsional effects.

The values to be substituted in Eq. 26 for the unit vertical influence load are of interest because they show the contribution of each deflection to the total. Table 13 gives these values, as well as values for H, by the simplified method, for the unit vertical influence load, for earth pressure from the left, and for temperature change.

TABLE 13.—SOLUTION OF EQ. 26

Equation	Points[a]								Earth[b] pressure from left	Temperature[b]
	3	4	5	6	7	8	9[c]	10		
					(*a*) ANALYSIS OF BRIDGE A					
$+0.0152074\ \delta_x$	0.0190	0.0692	0.1355	0.2092	0.2837	0.3501	0.3993	0.4249	132.92	21.21
$+0.0158529\ \delta_z$	−0.0000	−0.0002	−0.0013	−0.0033	−0.0060	−0.0092	−0.0120	−0.0136	−0.05	22.12
$-0.0486225\ \phi_y$	0.0025	0.0088	0.0171	0.0263	0.0353	0.0432	0.0489	0.0518	16.74	0.00
R_x	0.0215	0.0778	0.1513	0.2322	0.3130	0.3841	0.4362	0.4631	149.61	43.33
H	0.0216	0.0780	0.1516	0.2321	0.3118	0.3831	0.4317	0.4576	150.3	42.6
					(*b*) ANALYSIS OF BRIDGE B[d]					
R_x	0.0149	0.0587	0.1336	0.2215	0.3064	0.3759	0.4193	.....	123.88	30.63
H	0.0149	0.0587	0.1336	0.2214	0.3059	0.3746	0.4176	.....	124.0	30.4

[a] The points are equidistant and numbered from point 1 at the footing to point 10 at the crown. For points 1 and 2 the values are all zero since they are on the vertical part of the leg. [b] Values in kips. [c] Crown point for bridge B. [d] The details of the analysis for bridge B were the same as for bridge A and only the results are recorded.

It is evident that, except for temperature change, δ_x is the predominant factor contributing to the reaction R_x. The other two values are not only of different order but are always of opposite sign. The close agreement in the

data is added proof that the conclusion applies as well to elliptical skewed arches. The author has shown the proof that this conclusion is exact for flat-decked structures where the legs are vertical and the deck has a constant height.

The writer wishes to emphasize the manner in which Mr. Hodges has utilized this conclusion in his proposed simplified analysis. In step 1, the right arch or frame is analyzed for the redundant horizontal reaction, choosing as a base structure a statically determinate system, usually a simple span. This involves one unknown, H, and the solution of one elastic equation. In step 2, the final structure is analyzed using the foregoing system, with H and ϵH applied, as the base frame. Particular attention is called to the fact that the base frame in step 2 of the analysis is a statically indeterminate system; but it must be a system in equilibrium. Therefore, after H has been found and applied the second base system must be stabilized by the application of equilibrating reactions. This is possible only because the two elastic systems can be considered independently. In the analysis of bridge A the base structure chosen was a simple span with a rigid bracket from the hinge (on rollers) to the elastic centroid. For a symmetrical structure this centroid is on the center line. This procedure incorporates the desirable features of symmetry and agrees with the base structure ordinarily used in analyzing rectangular frames and arches—namely, a simple span. The writer believes this is another short cut that can be incorporated advantageously into the proposed simplified method.

The design of a skewed concrete rigid frame or arch bridge by the exact method is seldom attempted without the use of short cuts. Approximations should be accepted only where the limits of accuracy are understood and known. Any designer who has waded through this exacting work and the lengthy computations will appreciate any method that reduces the amount of computing or decreases the probability of error without sacrificing safety or economy. The proposed simplified method of analysis does this.

Comparisons, such as the foregoing, indicate that the approximation resulting from the neglect of the effect of the torsional elastic system on the rectangular elastic system is of the same order as the approximation usually made by neglecting shearing distortions when analyzing rectangular frames or arch bridges. The neglect of rib shortening due to axial thrust also falls in this category, except for flat arches.

Furthermore, in the solution of the simultaneous equations, analysis by the exact method encounters the troublesome problem of small differences. Only after considerable experience and numerous designs is it possible to know how many significant places are required in any particular case. Usually, therefore, the tendency is to carry the calculations to too many significant places. By using the proposed simplified method of analysis, this difficulty is overcome to a great extent. The problem is nonexistent in determining the horizontal rectangular reactions in step 1, since the solution of only one elastic equation is involved. In step 2, the number of simultaneous equations is reduced, and the occurrence of small differences is minimized.

Another interesting feature of this paper is its solution of the problem of foundation restraints. Rotation of the foundation because of compressibility

of the supporting material is an unknown factor and therefore analysis for boundary conditions (full fixity and free rotation) is justified. Although the added refinement of including the soil modulus would show the effect of partial restraint, such an analysis would depend on the accurate determination of this modulus and its uniformity. For this reason the writer favors design for boundary conditions since the true condition must be somewhere between the limits. Rotation of the foundation about the vertical axis is usually impossible. Restraints due to friction and approach walls will prevent this rotation, and, even where the approach walls are separated by expansion joints, the foundations are usually poured monolithic.

Earth pressure, as a load, is much more important in a skewed structure than it is in a rectangular one. The analysis for this load is usually relegated to a minor rôle, but the author gives it some of the major consideration it deserves. In the proposed method step 1 is still the determination of the rectangular horizontal reaction, but step 2 is much more involved. The base frame for step 2 is again a statically indeterminate system with H (as found in step 1) applied at the hinges; and, exactly as before, this base frame must be stabilized by cross shear reactions before the computations for the torsional elastic effect of the structure are started. In previous methods this stabilizing cross shear reaction is not separated from the redundant, and the total is applied at the hinge. The author divides this stabilizing cross shear reaction into two parts, one applied at the hinge and the other applied along the back of the abutment. The first part is obviously the foundation restraint but the second part requires explanation. Since equivalent fluid pressure was assumed acting normal to the abutment, it seems logical that frictional resistance to distortion will develop along the back of the abutment, and that this part of the frictional equilibrating cross shear reaction cannot exceed the normal pressure times the coefficient of friction of earth on concrete. The author applies this part of the equilibrating cross shear reaction concentrated at the center of normal pressure. He then transforms it into a cross shear reaction applied at the hinge plus a moment about the x-axis. The net difference in results between the author's method and previous methods should be this additional moment of transformation. Since it is on the safe side to do so, this moment of transformation should be included in the analysis. There is also a transformation moment about the y-axis which has been neglected. The close agreement of R_x and H (see Table 13) for earth pressure from the left indicates that the basic conclusion stated in the beginning of this discussion is just as valid for horizontal loads as for vertical loads.

This simplified method of analysis may be applied to double-span skewed structures if the following logical deductions are accepted. The evidence shows that the two elastic systems of single-span skewed structures are practically independent for vertical and horizontal loads and equally independent of the shape of the structure. A double-span structure may be considered as a single span with an elastic bracket at the center, to the ends of which are applied forces equal to the redundant reactions. This logic may be extended to include three or more spans.

With all the available data, and this simplified method of analysis, there should be no necessity for avoiding the skewed barrel type concrete rigid frame or arch bridge.

PHILLIPS H. LOVERING,[29] ASSOC. M. AM. SOC. C. E.[29a]—The value to the profession of many papers in *Proceedings* is emphasized by predictions that the ideas and information advanced should prove important. The value of this paper does not rest upon prediction. Within nine months of its publication, the writer already had used this design method for four bridges and thereby saved at least seven days of design time on each bridge. The time required for checking the design is reduced about four days.

More important, for any method based on the trial of assumed sections, subsequent trial designs may sometimes have to be made; and, in the case of former procedure, the suitability of the sections tried could not be determined until the end of the design, whereas using Mr. Hodges' method the suitability of the sections is determined by the second day, or in about one tenth the time required by the former method. The value of dividing the design into two parts with the longer and subordinate part coming last is thus emphasized.

The writer has found that the difficulty of choosing correct sections increases with the skew, and is caused mostly by the high temperature stresses. As in other arches, the temperature stresses can be held to a minimum by choosing sections that have a moment of inertia just sufficient to satisfy the stress requirements throughout a large part of the arch. In applying this principle to the rigid frame bridge, it becomes evident that the $\frac{y^2}{I}$-values at the knee are so comparatively small that it is best to thicken the knee because, although this decreases these values still more, the reduction of moment obtained in the middle part of the arch allows a considerable reduction in I and a corresponding increase in the $\frac{y^2}{I}$-values. The difficulty of obtaining a smooth intrados curve under conditions of a thick knee and a thin section throughout the central part of the arch is partly overcome by using an ellipse instead of the arc of a circle for the intrados with possibly a short tangent at the crown. This expedient becomes valuable for skews of 45° and more. Lengthening the legs also somewhat lessens the temperature stresses.

The high shears at the bottom of the leg are probably transferred across the construction joint into the footing mostly by friction. However, at least a large part of this uncertainty is removed by placing V-notches across the joint, and, if the longitudinal key occupies the middle third, the transverse V-notches should be set in the footing on either side of the key rather than across the bottom.

[29] Structural Designer, W. Earl Andrews, New York, N. Y.

[29a] Received by the Secretary February 8, 1944.

JAROSLAV POLIVKA,[30] M. AM. SOC. C. E.[30a]—For some decades structural engineers and scientists[31,32,33,34,35,36] have grappled with the relatively simple problem of a rectangular slab or plate supported on two opposite edges and subjected to transverse loading—a problem that cannot be identified with the solution of a simple beam, due to the effect of Poisson's ratio. The problem of a similar plate, but one with skewed shape, has only recently been solved with any degree of exactitude.[37] These problems are much simpler than those involved in skewed frames and arches.

Structural engineers familiar with such analyses would be reluctant to apply any method which, in spite of the complex mathematics involved, yields only approximate results. Mr. Hodges is to be commended for simplifying the mathematical analysis and for bringing the subject for discussion. However, in the analysis of even less important structures, engineers are inclined to proceed with extreme caution, and in the matter of bridges they would seem certain to look with disfavor on a method of analysis which is only an approximation and which has not yet been corroborated satisfactorily by tests on models, and certainly not by tests on existing structures.

There are three possibilities for progress in the design of skewed frames and arches:

(1) To delve deeper into the study of the structural behavior and stress distribution in skewed structural members, developing an analysis on the basis of the exact theory of elasticity as it is used for plates and shells,[35] and checking such analysis with laboratory tests and field measurements;

(2) To use only the results of actual tests for the design of skewed frames and arches; and

(3) To discard the skewed slab in the design of frames and arches and to substitute another type of structure, more amenable to exact analysis, as, for example, frame girders combined with continuous slabs, or heavy wall girders combined with cylindrical or arched shells in reinforced concrete (which may even prove to be more economical).

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[30] Research Associate, Univ. of California; Cons. Engr., Kaiser Co., Berkeley, Calif.

[30a] Received by the Secretary February 2, 1944.

[31] "Contribution à l'étude de l'équilibre élastique d'une plaque rectangulaire mince," by E. Estanave, thesis presented to the Faculty of Sciences, Univ. of Paris, Paris, 1900, p. 295.

[32] "Partial Differential Equation of the Fourth Degree," by M. Coialowitch, St. Petersburg, 1902 (in Russian).

[33] "Sur l'intégration de l'équation relative à l'équilibre des plaques élastiques encastrées," by G. Lauricella, *Acta mathematica*, Stockholm, Vol. 32, 1903, p. 189.

[34] "Sur l'intégration de l'équations biharmoniques," by S. Zaremba, *Bulletin de Cracovie*, Vol. 20, 1908.

[35] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1940.

[36] "Die Formaenderungen und Spannungen von rechteckigen elastichen Platten," by A. Nádai, *Forschungsarbeiten*, Berlin, 1915, Vol. 170, p. 171.

[37] "Ueber die elastiche Deformation parallelogrammfoermiger Platten," by A. Anzelius, *Der Bauingenieur*, Vol. 20, No. 35/36, September 8, 1939, p. 478.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

GEOLOGY IN HIGHWAY ENGINEERING

Discussion

BY CARL B. BROWN

CARL B. BROWN,[4] ESQ.[4a]—The professions of engineering and geology will both benefit by a paper that calls attention to the many aspects of highway engineering in which the training and viewpoint of the geologist can be of service to the engineer. A list as extensive as that of Mr. Huntting could be readily developed for railroad engineering, flood control engineering, military engineering, and, in fact, any engineering field dealing with the surface configuration of the earth, earth materials, or dynamic geology.

Unfortunately, many engineers, seemingly, have not learned to appreciate the aid that can be rendered to them not only by geologists, but by soil scientists, climatologists, agronomists, and specialists in numerous other fields. If some engineers seem noncooperative, this may be due, in part, to the failure of geologists, figuratively, to "come down to earth" or, literally, to evaluate their "stock in trade" on the basis of what is important in terms of the life of engineering works.

Although most of the paper shows an excellent appreciation of engineering needs, under the heading "Desirability of Bridge Sites and Road Sites with Respect to Possible Changes of Stream Channels," the conventional, and not too applicable, geological interpretation of erosion cycles and causes of stream-channel changes is given as follows:

> "Climate and rock types and structures act to modify the effects of erosion. * * * Hardness and chemical reactivity of rocks have obvious effects upon erosion. * * * Rock structures act to control erosion and in so doing they exert considerable control over stream patterns. * * * The effects upon shifting of streams attributable to stage of erosion [youth, maturity, or old age] and to modifications imposed by climatic and rock structural conditions are recognizable in advance through geologic investigation."

NOTE.—This paper by Marshall T. Huntting was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by W. W. Crosby.

[4] Head, Sedimentation Section, SCS, U. S. Dept. of Agriculture, Washington, D. C.

[4a] Received by the Secretary January 26, 1944.

These statements all refer to processes and effects that act or result so slowly that they are generally of little importance in considering engineering structures expected to last 100 years or even 500 years.

Throughout most of the United States, especially in the humid sections, accelerated soil erosion is a far more critical factor in highway and bridge construction and maintenance than geologic structure or rock type. Many instances could be cited of stream channels that once ran over rock shoals with high banks sufficient to confine all but the largest flood flows, and that today are filled with sand to within a foot or two of the top of the banks. As a result, even moderate flows are causing severe bank erosion, washing out bridge approaches, and covering highways with sand and silt.

The streams of the Southern Piedmont, in a geological sense, are degrading. The region as a whole is in a stage of geologic maturity of the second or later cycle, being perhaps half finished with the process of downcutting from a former peneplain surface to a new peneplain level. Many of the streams today, however, are not degrading their beds but are rapidly aggrading, because of the overload of coarse sediment produced by widespread sheet erosion and severe gullying of the deep soil mantle—a result of the clearing and cultivation of the land. The rate of slope erosion has been drastically accelerated, and the response has been stream-channel and valley aggradation. The rate of acceleration, itself, has fluctuated widely during the past 150 years with the ups and downs of agriculture. The future channel regime at any point where a bridge is to be built will depend more on future land use and the extent of soil conservation in the watershed than on the relatively immutable geologic factors of rock type, structure, and climate.

Engineers will find considerable information of value on the effects of erosion on stream-channel changes in two technical bulletins issued in 1939 and 1940 by the U. S. Department of Agriculture (31)(32).[4b]

Bibliography—

(31) **"Principles of Gully Erosion in the Piedmont of South Carolina,"** by H. A. Ireland, C. F. S. Sharpe, and D. H. Eargle, *Technical Bulletin No. 633*, U. S. D. A., 1939.

(32) **"Some Principles of Accelerated Stream and Valley Sedimentation,"** by S. C. Happ, Gordon Rittenhouse, and G. C. Dobson, *Technical Bulletin No. 695*, U. S. D. A., 1940.

[4b] Numerals in parentheses, thus: (31), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

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DISCUSSIONS

MILITARY AIRFIELDS
A SYMPOSIUM

Discussion

BY THOMAS E. STANTON

THOMAS E. STANTON,[6] M. AM. SOC. C. E.[6a]—No one who has been close to the procedure for designing and constructing military airfields can fail to be impressed with the thoroughness with which the Engineering Division of the Corps of Engineers has explored the subject. The magnitude of the problem is more readily appreciated when it is realized that the Corps has been forced to anticipate the development of heavier and heavier planes without always having definite advice as to whether or not a given field, originally designed for light traffic, would be converted into a heavy bomber field as soon as completed. The cost is exorbitant and the work seriously delayed if all fields are to be constructed for heavy duty. However, a more serious condition arises when a field designed to handle light traffic is inadequate because military needs subsequently require a service too heavy for the constructed capacity.

As stated by Colonel Stratton, some of the factors that are of considerable importance in pavement design for primary highways are of negligible importance in runway pavements. For impact and loading frequency, the width of runway is insurance against too frequent or rapid repetition of loads. Furthermore, the relatively large tires on airplane landing gear provide a wider spread and usually a somewhat lower unit loading than vehicle truck tires, thereby permitting greater deflection of the pavement than can be tolerated in highway pavement construction.

On the other hand, the much greater total loading from the large bombers and transport planes can be charged on the adverse side of the ledger and, in the case of relatively weak subgrade soils, requires a much heavier base design than necessary for the heaviest loads permitted on any highway pavement.

NOTE.—This Symposium was published in January, 1914, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*. as follows: February, 1944, by James B. Newman, Jr.

[6] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

[6a] Received by the Secretary Jaunary 31, 1944.

Another important factor is the deflection induced by the warming of airplane engines. The assumed designed loadings are increased 25% for all areas subject to warming-up vibrational stresses. This is not a matter of serious concern to the highway engineer.

The heaviest loading permissible on state highways is 9,000 lb per wheel, whereas the designed loading for all but one of the classes of airfields is considerably greater than this amount. The only airfield loading comparable to highway loading is the Class IV field which is based on a design wheel load of 5,000 lb as against 15,000 lb for a Class III field (Table 1). Furthermore, single tires are used on airplanes of this loading, whereas most trucks carrying loads in excess of 6,000 lb to 7,000 lb per wheel have dual tires.

For the same total load, single-tired trucks have proved much more destructive than dual-tired trucks. Part of the reason for this is in all probability the more severe bending of the pavement surface for equivalent total deflections. With heavily loaded single tires, failure results under a relatively small number of load repetitions for deflections of as little as 0.03 in. to 0.04 in. Therefore, with no restriction for the loading on a single tire, except the permissive total load per wheel, designs should be based on permissible deflections of not more than 0.02 in. to 0.04 in., depending upon the flexibility of the base and pavement. However, because of wartime tire conservation measures, trucks have been forced to limit the load per inch of tire width to a much lower value than was prewar practice. Subsequent highway tests with dual tires, and tests with the larger airplane tires, have proved that substantially greater deflections (say, 0.1 in. for the wide airplane tires currently used on the larger airplanes) can be tolerated when the total load is spread over such a broad band. The permissible deflection is also dependent upon the number of maximum load repetitions during the normal life of the pavement.